

# Mark Scheme (Results)

Summer 2018

Pearson Edexcel International A Level In Core Mathematics C34 (WMA02/01)

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#### **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# **General Instructions for Marking**

- 1. The total number of marks for the paper is 125.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

#### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes...

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{\text{ will be used for correct ft}}$
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- C or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

# General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

#### 1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where  $|pq| = |c|$ , leading to  $x = \dots$   
 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$ 

#### 2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

### 3. Completing the square

Solving 
$$x^2 + bx + c = 0$$
:  $(x \pm \frac{b}{2})^2 \pm q \pm c$ ,  $q \ne 0$ , leading to  $x = \dots$ 

## Method marks for differentiation and integration:

## 1. Differentiation

Power of at least one term decreased by 1.  $(x^n \rightarrow x^{n-1})$ 

## 2. Integration

Power of at least one term increased by 1.  $(x^n \to x^{n+1})$ 

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### **Exact answers**

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

#### **Answers without working**

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1. (i)	$\left\{ \int \frac{2x^2 + 5x + 1}{x^2}  \mathrm{d}x = \right.$	$\int 2 + \frac{5}{x} + \frac{1}{x^2} dx$	
		At least one of either $\pm \frac{A}{x} \to \pm \alpha \ln kx$ or $\pm \frac{B}{x^2} \to \pm \beta x^{-1}$ ; $A, B, \alpha, \beta$ non zero.	M1
	$=2x+5\ln kx-\frac{1}{x}\left\{+c\right\}$	At least 2 out of the 3 terms are correct. e.g. 2 of $2x, -\frac{1}{x}, 5 \ln kx$	A1
	Where $k \neq 0$ (k is usually 1)	$2x + 5 \ln kx - \frac{1}{x} \text{ with or without } + c \text{ all on}$ one line and apply isw once seen.  Do not allow $+ \frac{1}{-x} \text{ for } -\frac{1}{x}$	A1
		<i>A A</i>	[3]
	(i) Alternative l	· •	
	$\left\{ \int (2x^2 + 5x + 1)x^{-2}  dx = -\frac{1}{x}(2x) \right\}$	$\left\{ -\frac{1}{x}(4x+5) dx \right\}$	
		At least one of either $\pm \frac{A}{x} \rightarrow \pm \alpha \ln kx$ or	M1
	$=-2x-5-\frac{1}{c}+4x+5\ln kx \ \{+c\}$	$\pm \frac{B}{x^2} \rightarrow \pm \beta x^{-1}$ ; $A, B, \alpha, \beta$ non zero.	
	$= -2x - 5 - \frac{1}{x} + 4x + 5\ln kx \ \left\{ + c \right\}$	$\pm \frac{B}{x^2} \to \pm \beta x^{-1}; A, B, \alpha, \beta \text{ non zero.}$ At least 2 out of the 3 terms are correct.  At least 2 of $2x, -\frac{1}{x}, 5 \ln kx$ $2x - 5 - \frac{1}{x} + 5 \ln kx \text{ with or without } + c$	A1

	(i) Alternative by parts II:		
	$\left\{ \int (2x^2 + 5x + 1)x^{-2} dx = x^{-2} \left( \frac{2x^3}{3} \right)^{-2} dx \right\}$	$+\frac{5x^2}{2} + x + \int 2x^{-3} \left( \frac{2x^3}{3} + \frac{5x^2}{2} + x \right) dx$	
	$= \frac{2x}{3} + \frac{5}{2} + \frac{1}{x} + \frac{4x}{3} + 5\ln kx - \frac{2}{x} \left\{ + c \right\}$	At least one of either $\pm \frac{A}{x} \rightarrow \pm \alpha \ln k$ . $\pm \frac{B}{x^2} \rightarrow \pm \beta x^{-1}$ ; $A, B, \alpha, \beta$ non zero	M1
	3 2 x 3 x \( \frac{1}{2} \)	At least 2 out of the 3 terms are correct At least 2 of $2x$ , $-\frac{1}{x}$ , $5 \ln kx$	ct.
	$= 2x + \frac{5}{2} - \frac{1}{x} + 5 \ln kx \ \left\{ + c \right\}$ Where $k \neq 0$ (k is usually 1)	$2x + \frac{5}{2} - \frac{1}{x} + 5 \ln kx \text{ with or without } -\frac{1}{x}$ or $2x + 5 \ln kx - \frac{1}{x}$ with or without + all on one line and apply isw once see Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	c A1
	(i) Altamas	•	
$\left\{ ,\right\}$	(i) Alternative: $\left\{ \int \frac{2x^2 + 5x + 1}{x^2} dx = \int 2 + \frac{5x + 1}{x^2} dx = \int 2 + (5x + 1)x^{-2} dx \right\} = 2x - \frac{1}{x}(5x + 1) + \int \frac{5}{x} dx$		
		At least one of either $\pm \frac{A}{x} \rightarrow \pm \alpha \ln kx$ or $\pm \frac{B}{x^2} \rightarrow \pm \beta x^{-1}$ ; $A, B, \alpha, \beta$ non zero.	M1
	$=2x-5-\frac{1}{x}+5\ln kx  \{+c\}$	At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5 \ln kx$	A1
		$2x - 5 - \frac{1}{x} + 5 \ln kx \left\{ + c \right\}$ with or without + or $2x + 5 \ln kx - \frac{1}{x}$ with or without + c all on one line and apply isw once seen.  Do not allow + $\frac{1}{-x}$ for $-\frac{1}{x}$	c A1

	T			T
(ii)		$\left\{ \mathbf{I} = \int x \cos 2x  \mathrm{d}x  \right\} ,  \begin{cases} u = x \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos x \end{cases}$	$\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ $82x \Rightarrow v = \frac{1}{2}\sin 2x$	
			$\pm \lambda x \sin 2x \pm \mu \int \sin 2x \{dx\}$ BUT if the parts formula is quoted incorrectly score M0	M1
		$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \left\{ dx \right\}$	$\frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \left\{ dx \right\}$ simplified or un-simplified	A1
		$=\frac{1}{r}\operatorname{rein} 2r + \frac{1}{r}\operatorname{cos} 2r + c$	$\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x \text{ with or without } + c,$ $\frac{1}{2}x\sin 2x - \left(-\frac{1}{4}\cos 2x\right) \text{ is A0}$	A1
				[3]
				6
			on 1 Notes	
	Note		Forms e.g. $5\ln 5x$ or $2.5\ln x^2$ etc. and allow mod	ulus signs
(i)	Note	e.g. $5\ln kx $ There are no marks for attempts at $\frac{\int 2x^2}{x^2}$	$\frac{x^2 + 5x + 1  \mathrm{d}x}{\int x^2  \mathrm{d}x}$	
(ii)	Note	There are no marks for attempts at $\int x \cos x$	$\cos x  dx$	

Question Number	Scheme	Notes	Marks
2.	$x = \frac{3}{2}t - 5$ , $y = 4 - \frac{6}{t}$ , $t \neq 0$		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3}{2},  \frac{\mathrm{d}y}{\mathrm{d}t} = 6t^{-2}$	Both $\frac{dx}{dt} = \frac{3}{2}$ or $\frac{dt}{dx} = \frac{2}{3}$ and $\frac{dy}{dt} = 6t^{-2}$ $\frac{dy}{dt}$ can be simplified or un-simplified.  Note: This mark can be implied.	B1
	So, $\frac{dy}{dx} = \frac{6t^{-2}}{\left(\frac{3}{2}\right)} \left\{ = 4t^{-2} \right\}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\left\{\text{When } t = 3, \right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{9}$	4 9	A1 cao
			[3]
(b)	• $t = \frac{x+5}{\left(\frac{3}{2}\right)} \implies y = 4 - \frac{6}{\left(\frac{x+5}{\left(\frac{3}{2}\right)}\right)}$	An attempt to eliminate <i>t</i> .	M1
	• $t = \frac{6}{4 - y} \implies x = \frac{3}{2} \left(\frac{6}{4 - y}\right) - 5$ • $\frac{6}{4 - y} = \frac{2}{3} (x + 5)$	Achieves a correct equation in $x$ and $y$ only.	A1 o.e.
	$\Rightarrow y = 4 - \frac{9}{x+5}$		
	$\Rightarrow y = \frac{4(x+5)-9}{x+5}$		
	$\Rightarrow y = \frac{4x + 11}{x + 5}$	$\underline{a=4}$ and $\underline{b=11}$ or $\frac{4x+11}{x+5}$	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$ , $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
	Alternative 1	· ·	
	$y = \frac{ax + b}{x + 5} \Longrightarrow 4 - \frac{6}{t} =$	$=\frac{a(1.5t-5)+b}{}$	
	$y - \frac{1}{x+5} \rightarrow 4 - \frac{1}{t}$	1.5t - 5 + 5	
	$\Rightarrow 4 - \frac{6}{t} = \frac{1.5at - 5a + b}{1.5t} \Rightarrow 6t - 9 = 1.5at - 5a + b$ $\Rightarrow 6t = 1.5at \text{ or } -9 = -5a + b$	Substitutes for x and y and "compares coefficients" for term in t or constant term	M1
	a = 4 or $b = 11$	Correct value for a or b	A1
	a = 4 and $b = 11$	Correct values for a and b	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$ , $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
			7

		Alternative 2 for (b):			
	$y = \frac{4t - 6}{t} = \frac{3(4t - 6)}{2\frac{3t}{2}} = \frac{3(4t - 6)}{2(x + 5)} = \frac{4 \times \frac{3t}{2} - 9}{(x + 5)} = \frac{4(x + 5) - 9}{(x + 5)}$ M1: Obtains y in terms of x A1: Correct unsimplified expression				
		$\Rightarrow y = \frac{4x + 11}{x + 5}$ $\underline{a = 4} \text{ and } \underline{b = 11} \text{ or } \frac{4x + 11}{x + 5}$	A1		
		Do not isw so if they have $x \neq -5$ , $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1		
			[4]		
		Question 2 Notes			
<b>2.</b> (a)	Note	Note M1 can also be obtained by substituting $t = 3$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.			
	Note	Some candidates may use the Cartesian form in (a) possibly having done (b) find $y = \frac{4x+11}{x+5} \Rightarrow \frac{dy}{dx} = \frac{4(x+5)-4x-11}{(x+5)^2} \left( = \frac{9}{(x+5)^2} \right)  t = 3 \Rightarrow x = \frac{9}{2} - 5 = -\frac{1}{2} = \frac{4}{9}$ This would require a complete method to find the Cartesian equation and then B1 for derivative. Then M1 for a complete method attempting the derivative and substitution and A1 for 4/9 as in the main scheme.  The marks for obtaining the Cartesian equation can score in (b) provided their Cartesian is seen or used in (b). (i.e. if they do (a) first)	$\frac{1}{2}$ or the correcting for x or t		
		(1) (1) (1) (1)			

Question Number	Scheme		Notes	Marks	
3.	$f(x) = 2^{x-1} - 4 + 1.5x, x \in \mathbb{F}$	$\mathbb{R}; \qquad x_{n+1} = \frac{1}{3} \left( \frac{1}{3} \right)$			
(a)	$0 = 2^{x-1} - 4 + 1.5x \implies 1.5x = 4 - 2^{x-1} \text{ or } 4$	Sets $f(x) = 0$ and makes $1.5x$ (or $kx$ ) the subject of the formula using correct processing so allow sign errors only.	M1		
	$\Rightarrow x = \frac{2}{3} (4 - 2^{x - 1}) \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ or $\Rightarrow x = \frac{(4 - 2^{x - 1})}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$ $\Rightarrow x = \frac{4 - 2^{x - 1}}{1.5} \Rightarrow x = \frac{1}{3} (8 - 2^{x}) \text{ (*)}$				
	Special case: Starts with $1.5x = 4 - 2^{x-1}$ a	nd completes m	nethod with no $f(x) = 0$ is M1A0		
				[2]	
	Alternative wo	rking backwar	ds:		
	$x = \frac{1}{3}(8 - 2^{x}) \Rightarrow 3x = 8 - 2^{x} \Rightarrow 2^{x} - 8 + 3x = 0$ Multiplies by 3 and collects terms to one side or collects terms to one side and multiplies by 3				
	$2^{x} - 8 + 3x = 0 \Longrightarrow 2^{x-1} - 4 + 1.5x = 0$	: 0	Obtains $2^{x-1} - 4 + 1.5x = 0$ by cso.	A1	
	,			[2]	
(b)	$\lambda_1 = \frac{1}{2} \left( 0 - \lambda_1 \right)$		$x_0 = 1.6 \text{ into } \frac{1}{3} (8 - 2^{x_0}).$ be implied by $x_1 = \text{awrt } 1.66$	M1	
	$x_1 = 1.656$ , $x_2 = 1.616$		<b>and</b> $x_2 = \text{awrt } 1.616$	A1	
	$x_3 = 1.645$	$x_3 = 1.645$ only	(not awrt)	A1 cao	
	Mark their values in the order given i.e.	assume their f	irst calculated value is $x_1$ etc.		
(c)	f(1,6225) = 0,00100005			[3]	
(c)	f(1.6325) = -0.00100095 or awrt $-1 \times 10^{-3}$ f(1.6335) = 0.00157396 or awrt $1 \times 10^{-3}$ or awrt $2 \times 10^{-3}$ Chooses a suitable interval for $x$ , which is within $1.633 \pm 0.0005$ and either side of $1.63288$ and attempts to evaluate $f(x)$ for both values.			M1	
	Sign change (negative, positive) (and $f(x)$ is continuous) therefore root $(\alpha = 1.633)$		s correct awrt (or truncated) hange and a conclusion	A1 cso	
				[2]	
				7	

	Question 3 Notes				
<b>3.</b> (a)	M1	There are other methods for obtaining the printed equation but the M1 scores for setting $f(x) = 0$ and making $kx$ the subject of the formula using correct processing e.g. $0 = 2^{x-1} - 4 + 1.5x \Rightarrow \frac{2^x}{2} - 4 + 1.5x = 0 \Rightarrow 3x = 8 - 2^x \text{ M1}$ $\Rightarrow x = \frac{1}{3} \left( 8 - 2^x \right) \text{ (*)} \text{ A1}$ $0 = 2^{x-1} - 4 + 1.5x \Rightarrow 2^x - 8 + 3x = 0 \Rightarrow 3x = 8 - 2^x \text{ M1}$ $\Rightarrow x = \frac{1}{3} \left( 8 - 2^x \right) \text{ (*)} \text{ A1}$			
<b>3.</b> (c)	A1	Correct solution only. Candidate needs to state both of their values for $f(x)$ to awrt (or truncated) 1sf along with a reason and conclusion. Reference to change of sign or $f(1.6325) \times f(1.6335) < 0$ or a diagram or $< 0$ and $> 0$ or one positive, one negative are sufficient reasons. There must be a conclusion, e.g. $\partial = 1.633(3 \text{ dp})$ . Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion could be "change of sign, so true" In part (c), candidates can construct their proof using a narrower range than [1.6325, 1.6335] which contains the root 1.632888767			

Question Number		Scheme		Notes	Marks
<b>4.</b> (a)	(1+px)	$e^{-4} = 1 + (-4)(px) + \frac{(-4)(-5)}{2!}(px)^2 + \frac{(-4)(-5)}{3!}$	$\frac{(-4)(-5)(-6)}{3!}(px)^3 + \dots$ see notes		
		$= 1 - 4px + 10p^2x^2 - 20p^3x^3 + \dots$	Three of the four simplified.	ir terms correct and	A1
		or = $1 - 4(px) + 10(px)^2 - 20(px)^3 +$		orrect and simplified and ect answer is seen. part (a).	A1
					[3]
(b)		$\left\{ f(x) = \frac{3+4x}{(1+px)^4} = \right\} (3+4x)(1-4px + 1)$	$10p^2x^2 - 20p^3x^3$	+) =	M1
	There	Attempts to expand $(3 + 4x) \times$ the should be evidence of at least $(3 \times $ one term from			
		<b>Note:</b> $f(x) = 3 + (4 - 12p)x + (30p^2 - 10p^2 - 10p$	$(6p)x^2 + (40p^2 - 6p)x^2$	$(50p^3)x^3 +$	
	= 3 -	$\frac{12px + 30p^2x^2}{\Rightarrow} - 60p^3x^3 + 4x - 16px^2 + 40p^2$ $\Rightarrow$ $"30p^2 - 16p" = 2"(4 - 12p)"$	mark Multip	on the previous M blies out to give exactly x and exactly 2 terms in apts one coefficient =	dM1
		Or or $2"(30p^2 - 16p)" = "(4 - 12p)"$	implied by la	er. This mark can be atter working. Allow x's for this mark	
		$30p^2 - 16p = 2(4 - 12p)$	Correct equation	n with no x's	A1
		$30p^{2} + 8p - 8 = 0$ $(p-4)(3p+2) = 0 \text{ or } (5p-2)(6p+4) = 0 \implies p$ or $(p^{2} + 4p - 4 = 0 \implies (5p-2)(3p+2) = 0 \implies p = \dots$	Correct me leading to a If working guidance for working is	t on the 1st M mark thod for solving a 3TQ at least one value. is shown see general or solving 3TQs. If no shown then you may ack to see if their 3TQ	dM1
	<	$p = \frac{2}{5}, -\frac{2}{3} \Rightarrow \text{As } p > 0, \text{ then}$ $p = \frac{2}{5}$	$p = \frac{2}{5}$ only.		A1
					[5]
(c)		$40\left(\frac{2}{5}\right)^2 - 60\left(\frac{2}{5}\right)^3$	their coefficient o	$p = \frac{2}{5}$ from part (b) into of $x^3$ (which comes from om their expansion)	M1
		Coefficient of $x^3$ is $\frac{64}{25}$	Allow $\frac{64}{25}x^3$ , $2\frac{14}{25}$	$\frac{4}{5}$ . Condone 2.56. $\frac{4}{5}x^3$ , 2.56 $x^3$ are offered, score <b>A0</b>	A1
					[2]
			4 NI - 4 -		10
<b>4.</b> (a)	M1	Question Uses the binomial expansion with $n = -4$ and	4 Notes $1'x' = px.$		
<b>4.</b> (a)	Note   Uses the binomial expansion with $n = -4$ and $x = px$ .   Note   M1 can be given for either $1 + (-4)(px)$ or $\frac{(-4)(-5)}{2!}(px)^2$ or $\frac{(-4)(-5)(-6)}{3!}(px)^3$				
(b)	Note	Allow recovery in part (b) from missing brack	tets in part (a). e.g.	$px^2$ now becoming $p^2x$	<sup>2</sup> .

Question Number	Scheme	Notes	Ma	ırks
5.	$f: x \to e^{2x} - 5, x \in \mathbb{R}; g:$	$x \to \ln(3x-1), x \in \mathbb{R}, x > \frac{1}{3}$		
(i) (a)	$y = e^{2x} - 5 \implies x = e^{2y} - 5$ $x + 5 = e^{2y} \implies \ln(x+5) = 2y$	Attempt to make <i>x</i> (or swapped <i>y</i> ) the subject using correct processing so allow sign errors only.	M1	
	$(y =) \frac{1}{2} \ln(x+5) \left\{ (f^{-1} : x \to) \frac{1}{2} \ln(x+5) \right\}$	$\frac{1}{2}\ln(x+5)$ or $\frac{1}{2}\ln x+5 $ or $\ln(x+5)^{\frac{1}{2}}$ . Correct expression ignoring how it is referenced <b>but must be in terms of </b> $x$ <b>.</b> Do not allow $\ln(x+5) \cdot \frac{1}{2}$ or e.g. $\ln x + 5$ or $\ln(x+5)$ unless the correct answer is seen previously or subsequently.	A1	
	Domain: $x > -5$ or $(-5, \infty)$	$x > -5$ or $(-5, \infty)$ Condone domain $> -5$	B1	
(b)	$fg(3) = e^{2\ln(3(3)-1)} - 5$ (NB fg(x) = 9x <sup>2</sup> - 6x - 4)	g goes into f and $x = 3$ is substituted into the result or finds $g(3) \{= \ln 8\}$ and substitutes into f	M1	[3]
	$\left\{ = e^{2\ln 8} - 5 = 64 - 5 \right\} = 59$	59 cao	A1	
	, ,			[2]
(ii)(a)	y <b>A</b>	A vishape with the vertex on the positive x-axis (with no significant asymmetry about the vertical through the vertex). The left hand branch must extend into the second quadrant. Do not allow a "y" shape unless the part below the x-axis is dotted or "crossed out"	B1	
	$\frac{1}{4}a$	States $(0, a)$ and $(\frac{1}{4}a, 0)$ or $\frac{1}{4}a$ marked in the correct position on the $x$ -axis and $a$ marked in the correct position on the $y$ -axis. Other points marked on the axes can be ignored.	В1	[2]
(b)	${4x - a = 9a \Rightarrow}  x = \frac{10a}{4}  {\text{or } x = \frac{5a}{2}}$	$x = \frac{10a}{4} \text{ or } x = \frac{9a+a}{4} \text{ or } x = \frac{5a}{2}$ (may be implied)	B1	[2]
	-(4x - a) = 9a or $4x - a = -9a$	Attempt at the "second" solution. Accept $-(4x - a) = 9a$ or $4x - a = -9a$ or $-4x = 8a$ . Do not condone (unless recovered) invisible brackets in this case.	M1	
	x = -2a	x = -2a	A1	
	$\left\{x = \frac{5}{2}a \Rightarrow\right\} \left \frac{5}{2}a - 6a\right  + 3\left \frac{5}{2}a\right ; = 11a$	Substitutes at least one of their $x$ values from solutions of $\begin{vmatrix} 4x - a \end{vmatrix} = 9a$ where $x < 6a$ into $\begin{vmatrix} x - 6a \end{vmatrix} + 3 \begin{vmatrix} x \end{vmatrix}$ and finds at least one value for $\begin{vmatrix} x - 6a \end{vmatrix} + 3 \begin{vmatrix} x \end{vmatrix}$	M1	
	${x = -2a \Rightarrow}  -2a - 6a  + 3 -2a ; = 14a$	Must apply the modulus.  Both 11a and 14a and no other answers	A1	
		The state of the s		[5] 12

		Question 5 Notes
(b)	Note	The values of x might be found by squaring: $ 4x - a  = 9a \Rightarrow 16x^2 - 8ax + a^2 = 81a^2 \Rightarrow 16x^2 - 8ax - 80a^2 = 0$ $16x^2 - 8ax - 80a^2 = 0 \Rightarrow x = \frac{5a}{2}, -2a$ Score as follows: B1 for a correct 3 term quadratic (terms collected after squaring) M1: Solves their 3 term quadratic (usual rules)
		A1: $x = \frac{5a}{2}, -2a$

Question Number	Scheme		Notes	
6.	$\sqrt{5}\cos q$ -	$2\sin q$	$\circ R\cos(q+a)$	
(a)	$R = 3$ $R = 3$ , cao (±3 is B0) ( $\sqrt{9}$ is B0)		$R = 3$ , cao (±3 is B0) ( $\sqrt{9}$ is B0)	B1
	$\tan \alpha = \pm \frac{2}{\sqrt{5}}, \ \tan \alpha = \pm \frac{\sqrt{5}}{2} \implies \alpha = \dots$ (Also allow $\cos \alpha = \pm \frac{\sqrt{5}}{3}$ or $\pm \frac{2}{3}$ , $\sin \alpha = \pm \frac{2}{3}$ or $\pm \frac{\sqrt{5}}{3} \implies \alpha = \dots$ , where "3" is their <i>R</i> .)		M1	
	$\alpha = 0.7297276562 \Rightarrow \alpha = 0.7297 $ (4 sf		Anything that rounds to 0.7297 (Degrees is 41.81 and scores A0)	A1
	{Note: $\sqrt{5}\cos q$	- 2sin q	$7 = 3\cos(q + 0.7297)\}$	[3]
(b)	$\sqrt{5}$ co	os q - 2s	$\sin q = 0.5$	
		Attemp	pts to use part (a) "3" $\cos(\theta \pm "0.7297") = 0.5$	
	$3\cos(\theta + 0.7297) = 0.5$	9and p	proceeds to $\cos(\theta \pm 0.7297) = K$ , $ K  < 1$	
	0.5	May b	e implied by $\theta \pm "0.7297" = 1.4033$	M1
	$\Rightarrow \cos(\theta + 0.7297) = \frac{0.5}{3}$	or $\theta \pm$	$\pm "0.7297" = \cos^{-1}\left(\frac{0.5}{\text{their }3}\right) (=1.4033)$	
	$\theta_1 = 0.673648 \Rightarrow \theta_1 = 0.674 (3 \text{ sf})$	$48 \Rightarrow \theta_1 = 0.674 (3 \text{ sf})$ Anything that rounds to 0.674		A1
	$\theta_2 + "0.7297" = "-1.4033" \Rightarrow \theta_2 = \dots$	<b>dependent on the previous M mark</b> Correct attempt at a second solution in the range. Usually given for: $\theta_2$ + their 0.7297 = - their 1.4033 $\Rightarrow \theta_2$ =		
	$\theta_2 = -2.133048 \Rightarrow \theta_2 = -2.13 (3 \text{ sf})$			
	For solutions in (b) that are otherwise f	fully con	rrect, if there are extra answers in the range, all A mark.	
	For candidates who work consistently in d	legrees	in (a) and (b) allow awrt 38.6° and awrt – 122° will be lost in part (a)	
	/ =			[4]
(c)	$f(x) = A\left(\sqrt{5}\cos\theta - 2s\right)$	$\sin\theta$ )+	$B, \theta \in \mathbb{R}; -15 \leqslant f(x) \leqslant 33$	
	$\Rightarrow$ $-15 \leqslant 3$	$4\cos(\theta -$	$+0.730) + B \leqslant 33$	
	Note that part (c)	is now 1	marked as B1M1A1A1	
	B=9		Correct value for B	B1
	$ \begin{bmatrix} 3A + B = 33 \\ -3A + B = -15 \end{bmatrix} $ or $ \begin{bmatrix} 3A + B = -1 \\ -3A + B = 33 \end{bmatrix} $	$\begin{bmatrix} RA + B = 33 \\ -RA + B = -15 \end{bmatrix}$ or $\begin{bmatrix} RA + B = -15 \\ -RA + B = 33 \end{bmatrix}$		M1
	A = 8 or $A = -8$		One correct value for A	A1
	A = 8 and $A = -8$		$(+0.730) + B \le 33$ marked as B1M1A1A1  Correct value for $B$ Writes down at least one pair of simultaneous equations (or inequalities) of the form $(RA + B = 33)$ $(-RA + B = -15)$ and finds at least one value for $A$ M1	
				[4]
				11

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(c) <b>Alt 1</b>		B = 9	Correct value for B	B1	
		(2)(A)(3) = 3315	$(2)(A)$ (their $R$ ) = 33 – -15 $\Rightarrow$ $A =$	M1	
		A = 8 or $A = -8$	One correct value for A	A1	
		A = 8 and $A = -8$	Both values correct	A1	
				[4]	
(c) Alt 2		$B = \frac{33 - 15}{2} = 9$	Correct value for B	B1	
		$3A = 33 - 9 \Rightarrow A = 8$	(their $R$ ) $A = 33$ – their $B \Rightarrow A =$	M1	
		A = 8 or $A = -8$	One correct value for A	A1	
		A = 8 and $A = -8$	Both values correct	A1	
				[4]	
	Question 6 Notes				
(c)	Note The M mark may be implied by correct answers so obtaining A = 8 implies M1A1				

Question Number		Scheme	Notes	Marks
7.		$V = \frac{1}{3}\rho h^2(90 - h) = 30\rho h^2 - h^2$	$-\frac{1}{3}\rho h^3; \frac{\mathrm{d}V}{\mathrm{d}t} = 180$	
		$\frac{dV}{dh} = 60\rho h - \rho h^2$	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h}=\right\}\pm ah\pm bh^2,\ a\neq 0,\ b\neq 0$	M1
		$dh = d\rho h - \rho h$	$60ph - ph^2$ Can be simplified or un-simplified.	A1
	$\left\{ \frac{\mathrm{d}V}{\mathrm{d}h} \times \right.$	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow \left\{ 60\rho h - \rho h^2 \right\} \frac{\mathrm{d}h}{\mathrm{d}t} = 180$	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 180$	
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\right\}$	$\left\{ \frac{V}{dt} \div \frac{dV}{dh} \Rightarrow \right\}  \frac{dh}{dt} = 180 \times \frac{1}{60\pi h - \pi h^2}$	or $180 \div \text{their } \frac{dV}{dh}$ This is for a correct <b>application</b> of the chain rule and not for just quoting a correct chain rule.	M1
	When $h = 15$ , $ \left\{ \frac{dh}{dt} = \right\} \frac{1}{60\rho(15) - \rho(15)^2} \times 180 \left\{ = \frac{4}{15\rho} \right\} $ $ \left\{ \frac{dh}{dt} = 0.0848826 \Rightarrow \right\} \frac{dh}{dt} = \underline{0.085} \text{ (cm s}^{-1}\text{) (2 sf)} $		Dependent on the previous M mark. Substitutes $h = 15$ into an expression which is a result of a quotient (or their rearranged quotient) of their $\frac{dV}{dh}$ and 180. May be implied by awrt 0.08 or 0.09.	dM1
			Awrt 0.085 or allow $\frac{4}{15\pi}$ oe (and isw if necessary)	A1 cao
				[5]
		A 14 4 M - 41 1 f 4	J 64 N. 1 A 1	5
		Alternative Method for t	)	
		Product rule: $\begin{cases} u = \frac{1}{3}\rho h^2 \\ \frac{du}{dh} = \frac{2}{3}\rho h \end{cases}$	$v = 90 - h$ $\frac{\mathrm{d}v}{\mathrm{d}h} = -1$	
	$\frac{dV}{dh} = \frac{2}{3}\rho h(90 - h) + \frac{1}{3}\rho h^{2}(-1)$ Can be $\frac{2}{3}\rho h(90 - h) + \frac{1}{3}\rho h^{2}(-1)$		$= \begin{cases} \pm \alpha h(90 - h) \pm \beta h^2(-1), & \alpha \neq 0, \beta \neq 0 \\ \text{e simplified or un-simplified.} \end{cases}$	M1
			$\frac{2}{3}\rho h(90 - h) + \frac{1}{3}\rho h^2(-1)$ Can be simplified or un-simplified.	
			n 7 Notes	
7.	Note	$\frac{dV}{dh}$ does not have to be explicitly stated f that they are differentiating their V.	or the 1st M1 and/or the 1st A1 but it should be	be clear
	Note		- h) scores M0A0 even though it satisfies the	ne

Question Number	Scheme	Notes	Marks	
8.	$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ l_2: \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}; \ \text{Let } \theta = \text{acute angle between } PQ \text{ and } l_1.$			
(a)	i: $1 + \lambda = 6 + \mu$ (1) j: $-3 + 2\lambda = 4 + \mu$ (2) k: $2 + 3\lambda = 1 - \mu$ (3)			
	$\mathbf{j}: -3 + 2\lambda$	$=4 + \mu$ (2)		
	$\mathbf{k}: 2+3\lambda =$	$=1-\mu$ (3)		
	(1) and (2) yields $l = 2$ , $m = -3$ (1) and (3) yields $l = 1$ , $l = -4$	Attempts to solve a pair of equations to find at least one of either $/ =$ or $m =$	M1	
	(2) and (3) yields $/ = 1.2$ , $m = -4.6$	/ and m are both correct	A1	
	Checking (3): $8 \neq 4$ Checking (2): $-1 \neq 0$	Attempts to show a contradiction	M1	
	Checking (1): $2.2 \neq 1.4$ $l_1$ and $l_2$ do not intersect.	Correct comparison and a conclusion. Accept "do not meet" and accept "are skew".  Requires all previous work to be correct.	A1	
	Allow a calculation that gives "8 = 4 so the lines do not meet"			
			[4]	
	Alternative	e for part (a):		
		Attempts to solve a pair of equations to find at least one of either $/ =$ or $m =$	M1	
		Shows any two of (1) and (2) yielding / = 2 (1) and (3) yielding / = 1		
	(1) and (2) yields $l = 2$ , $m = -3$ (1) and (3) yields $l = 1$ , $m = -4$ (2) and (3) yields $l = 1.2$ , $m = -4.6$	(2) and (3) yielding / = 1.2		
		<b>or</b> shows any two of	A1	
		<b>(1)</b> and <b>(2)</b> yielding $m = -3$		
		(1) and (3) yielding $M = -4$		
		(2) and (3) yielding $m = -4.6$		
	T 0 0 1	Attempts to show a contradiction	M1	
	E.g. So $2 \neq 1$ $l_1$ and $l_2$ do not intersect.	Correct comparison and a conclusion. Accept "do not meet" and accept "are skew".  Requires all previous work to be correct.	A1	
l		Acquires an previous work to be correct.	[4]	

(b)	$\overrightarrow{OP} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \ \overrightarrow{OQ} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$		
	$(\overrightarrow{DG})$ $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$	Full method of finding $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ where $P$ and $Q$ have been found by using $\lambda = 0$ in $l_1$ and $\mu = -1$ in $l_2$	M1
	$\left(\overrightarrow{PQ} = \right) \begin{pmatrix} 3\\3\\2 \end{pmatrix} - \begin{pmatrix} 1\\-3\\2 \end{pmatrix} = \begin{pmatrix} 4\\6\\0 \end{pmatrix} \text{ or } \left(\overrightarrow{QP} = \right) \begin{pmatrix} -4\\-6\\0 \end{pmatrix}$	Correct $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ . Also allow for direction, $\mathbf{d}_{PQ} = 2\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$ and allow coordinates e.g. $(4, 6, 0)$	A1
	$\mathbf{d}_{1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ \mathbf{d}_{PQ} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$	Realisation that the dot product is required between $\pm A \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	M1
	$\cos q = \pm \left( \frac{(1)(4) + (2)(6) + (3)(0)}{\sqrt{(1)^2 + (2)^2 + (3)^2} \cdot \sqrt{(4)^2 + (6)^2 + (0)^2}} \right)$	Dependent on the previous M mark. An attempt to apply the dot product formula between $\pm A \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	dM1
	$\cos \theta = \frac{16}{\sqrt{14}.\sqrt{52}} \Rightarrow \theta = 53.62985132 = 53.63 \text{ (2 dp)}$	Anything that rounds to 53.63	A1
			[5]

(c)	d .	et trigonometric equation involving $d$ . e.g. $\frac{d}{\text{their } PQ} = \sin q$ , o.e.	M1
	${d = \sqrt{52} \sin 53.63 \Rightarrow} d = 5.8064 = 5.81 (3sf)$	Anything that rounds to 5.81	A1
			[2]
	<b>Alternative for part (c):</b> (Let <i>M</i> be the po	pint on $l_1$ closest to $Q$ )	
	$\overrightarrow{OM} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow \overrightarrow{QM} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} \lambda - 4 \\ 2\lambda - 6 \\ 3\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0 \Rightarrow \lambda - 4 + 4\lambda - 12 + 9\lambda = 0$ $\begin{pmatrix} \lambda - 4 \\ 2\lambda - 6 \\ 3\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda - 4 + 4\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{8}{7}$ $\lambda = \frac{8}{7} \Rightarrow \overrightarrow{QM} = \frac{1}{7} \begin{pmatrix} -20 \\ -26 \\ 24 \end{pmatrix} \Rightarrow  \overrightarrow{QM}  = \frac{1}{49} \sqrt{20^2 + 26^2 + 24^2}$	Applies a complete and correct method that leads to an expression for the shortest distance	M1
	$=\sqrt{\frac{236}{7}} = 5.81$	Anything that rounds to 5.81	A1
			[2]
			11

Question Number		Scheme		Notes	Marks
9.		$f(x) = \frac{12}{(2x - 1)^2}$	1), 1	$\leqslant x \leqslant 5; \ y = \frac{4}{3}$	
	-1 $-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$		where	$u^{-2} \to \pm / (2x - 1)^{-1} \text{ or } \pm / u^{-1}$ $u = 2x \pm 1; \ \lambda \neq 0$	M1
(a)	$\int \int \overline{(2x)^2}$	$\left. \frac{1}{(-1)^2}  \mathrm{d}x \right\} = \frac{(2x-1)^{-1}}{(-1)(2)} \left\{ +c \right\}$		$\frac{(-1)^{-1}}{(2)}$ or $-\frac{1}{2(2x-1)}$ oe with or without $+c$ .	A1
			Can be	simplified or un-simplified.	[2]
(b)		$\rho \int \left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$	Ignore	$\int \left(\frac{12}{2x-1}\right)^2 dx \text{ or } \pi \int \frac{144}{(2x-1)^2} dx$ limits and dx. The implied and the $\pi$ may be recovered later.	B1
		V <sub>1</sub> = 1		$\frac{-1}{(2x-1)} \bigg]_{1}^{5}$	
		$\begin{pmatrix} \begin{pmatrix} -1 \end{pmatrix} \end{pmatrix} \begin{pmatrix} -1 \end{pmatrix} \end{pmatrix}$	form =	s x-limits of 5 and 1 to an expression of the $-\beta(2x-1)^{-1}$ ; $\beta \neq 0$ and subtracts the correct und.	M1
	$=144(\pi)$	$\left( \left\lfloor \frac{2(2(5)-1)}{2(2(1)-1)} \right\rfloor - \left\lfloor \frac{2(2(1)-1)}{2(2(1)-1)} \right\rfloor \right)$	or with	t expression for the integrated volume with tout the $\pi$ . simplified or un-simplified. implied by 64 or 64 $\rho$ .	A1
		$\left\{ = -72(\pi$			
	Note: $\pi$			ted directly as $64\pi$ or $64$ with no incorrect	
		working seen scores M		esumably on a calculator)	
			and h	ots to use the formula $\rho r^2 h$ with numerical $r$ with at least one of $r = \frac{4}{3}$ or $h = 4$ correct	M1
	${V_{\text{cylinder}}} = \rho \left(\frac{4}{3}\right)^2 (4) \left\{ = \frac{64}{9}\rho \right\}$			mpts $\pi \int_{1}^{5} \left(\frac{4}{3}\right)^{2} dx$ or $\pi \int_{0}^{5} \left(\frac{4}{3}\right)^{2} dx$ t expression for $V_{\text{cylinder}}$	
				(4) or $\frac{64}{9}\rho$ implies this mark	A1
	$\left\{ \operatorname{Vol}(R)\right\}$	$(1 = 64p - \frac{64p}{9}) \Rightarrow Vol(R) = \frac{5}{2}$	$\frac{512}{9}\rho$	$\frac{512}{9}\rho \text{ or } 56\frac{8}{9}\rho$	A1
					[6] 8
			Questi	on 9 Notes	<u></u>
<b>9.</b> (b)	Note	See extra notes below for how t		attempts at $\pi \int_{1}^{5} \left( \left( \frac{12}{2x-1} \right) - \left( \frac{4}{3} \right) \right)^{2} dx$	
	Note	An acceptable approach is $\pi$	$\frac{1}{5}\left(\frac{12}{2x-1}\right)$	$\left(\frac{1}{1}\right)^2 - \left(\frac{4}{3}\right)^2 dx$	

Attempts at 
$$\pi \int_{1}^{5} \left( \left( \frac{12}{2x-1} \right) - \left( \frac{4}{3} \right) \right)^{2} dx$$
:

$$V = \pi \int_{1}^{5} \left( \frac{12}{2x - 1} - \frac{4}{3} \right)^{2} dx = \pi \int_{1}^{5} \left( \frac{144}{(2x - 1)^{2}} - \frac{32}{2x - 1} + \frac{16}{9} \right) dx$$

**B1** for the embedded  $\rho \int \left(\frac{12}{2x-1}\right)^2 dx$  ( $\pi$  may be recovered later)

$$= \pi \left[ -\frac{72}{2x-1} - 16\ln(2x-1) + \frac{16}{9}x \right]_{1}^{5}$$
$$= \pi \left[ \left( -\frac{72}{9} - 16\ln 9 + \frac{80}{9} \right) - \left( -72 + \frac{16}{9} \right) \right]$$

M1A1 for the embedded  $-\frac{72}{9} - (-72)$  or  $\left(-\frac{72}{9} - (-72)\right)\pi$   $\left(=\frac{640}{9}\pi - 48\ln 9\right)$ 

$$V = \pi \int_{1}^{5} \left( \frac{12}{2x - 1} - \frac{4}{3} \right)^{2} dx = \pi \int_{1}^{5} \left( \frac{144}{(2x - 1)^{2}} + \frac{16}{9} \right) dx$$

**B1** for the embedded  $\rho \int \left(\frac{12}{2x-1}\right)^2 dx$  ( $\pi$  may be recovered later)

$$= \pi \left[ -\frac{72}{2x - 1} + \frac{16}{9} x \right]_{1}^{9}$$
$$= \pi \left[ \left( -\frac{72}{9} + \frac{80}{9} \right) - \left( -72 + \frac{16}{9} \right) \right]$$

**M1A1** for the embedded  $-\frac{72}{9}-(-72)$  or  $\left(-\frac{72}{9}-(-72)\right)\pi$   $\left(=\frac{640}{9}\pi\right)$ 

$$V = \pi \int_{1}^{5} \left( \frac{12}{2x - 1} - \frac{4}{3} \right)^{2} dx = \pi \int_{1}^{5} \left( \frac{144}{(2x - 1)^{2}} - \frac{16}{9} \right) dx$$

**B1** for the embedded  $\rho \int \left(\frac{12}{2x-1}\right)^2 dx$  ( $\pi$  may be recovered later)

$$= \pi \left[ -\frac{72}{2x - 1} - \frac{16}{9} x \right]_{1}^{5}$$
$$= \pi \left[ \left( -\frac{72}{9} - \frac{80}{9} \right) - \left( -72 - \frac{16}{9} \right) \right]$$

**M1A1** for the embedded  $-\frac{72}{9} - (-72)$  or  $\left(-\frac{72}{9} - (-72)\right)\pi$   $\left(=\frac{512}{9}\pi\right)$ 

Question Number	Scheme			Notes	
10.		$C: xe^{5-2y} - y = 0 \text{ or } \ln x + 5$	$-2y - \ln y$	= 0; $P(2e^{-1}, 2)$ lies on $C$ .	
	Either		Obtains e	1 1	
	•	$e^{5-2y} - 2xe^{5-2y} \frac{dy}{dx} - \frac{dy}{dx} (=0)$		$\pm Bxe^{5-2y}\frac{dy}{dx} \pm \frac{dy}{dx} (=0)$	
	•	$e^{5-2y} - 2y\frac{dy}{dx} - \frac{dy}{dx} \Big( = 0 \Big)$		$\frac{dy}{dx} \pm By \frac{dy}{dx} \pm \frac{dy}{dx} (= 0)$ $\pm K \frac{dy}{dx} \pm \frac{B}{y} \frac{dy}{dx} (= 0)$	M1
	•	$\frac{1}{x} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} (=0)$		$= \pm A e^{\pm \alpha \pm 2y} \pm B y e^{\pm \alpha \pm 2y}$	
	•	$\frac{dx}{dy} = e^{2y-5} + 2ye^{2y-5}$		$= \pm B e^{\pm 2y} \frac{dy}{dx} \pm Ky e^{\pm 2y} \frac{dy}{dx}$	
	•	$e^5 = e^{2y} \frac{dy}{dx} + 2y e^{2y} \frac{dy}{dx}$	Correct d	0; $a$ , $b$ can be $0$ ifferentiation. The "= $0$ " may be y later work.	A1
		Ignore any " $\frac{dy}{dx}$ =" in f			
			Uses P(2	e <sup>-1</sup> , 2) and their gradient equation to	
	At $P$ , $e^{5-2(2)} - 2(2e^{-1})e^{5-2(2)}\frac{dy}{dx} - \frac{dy}{dx} = 0$ $\Rightarrow e - 4\frac{dy}{dx} - \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{e}{5}$			find a numerical value for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ . Could	
			have extra or fewer $\frac{dy}{dx}$ terms and may have		M1
			_	d their expression wrongly before	
				substituting. Accept $\frac{dy}{dx}$ = awrt 0.54 as evidence.	
	•	$\left\{ m_T = \frac{e}{5} \Rightarrow \right\}$ $y - 2 = \frac{e}{5} \left( x - \frac{2}{e} \right) \text{ or } x - \frac{2}{e} = 5e$		<b>Dependent on the previous M</b> mark. A correct attempt at an equation of the tangent at the point $P(2e^{-1}, 2)$ using their	<b>d</b> M1
	$ 2 = \frac{e}{5}(2e^{-1}) + c \implies c = \frac{8}{5} \implies y = $ $ y = 0 \implies -2 = \frac{e}{5}\left(x - \frac{2}{e}\right) \implies x = -\frac{8}{e} \iff A $		$\frac{e}{5}x + \frac{8}{5}$	numerical $\frac{dy}{dx}$ . If using $y = mx + c$ must reach as far as $c =$	
			$\left\{\left(-\frac{8}{e},0\right)\right\}$	Finds at least one correct intercept.	A1
	<i>x</i> = 0	$0 \Rightarrow y - 2 = \frac{e}{5} \left( -\frac{2}{e} \right) \Rightarrow y = \frac{8}{5}  \{ \Rightarrow B$	$\left\{0,\frac{8}{5}\right\}$	For $-\frac{8}{e}$ , allow awrt -2.94.	
			Dependent on both previous M marks.		
		Area $OAB = \frac{1}{2} \left( \frac{8}{e} \right) \left( \frac{8}{5} \right)$	Applies -	$\frac{1}{2}$ (their $x_A$ )(their $y_B$ ) where their $x_A$	<b>dd</b> M1
	2(e)(5)		and $y_B$ are negative a	e <b>exact</b> . Condone a method that gives a area.	
	$= \frac{32}{5e} \text{ or } \frac{32}{5} e^{-1}$		$\frac{32}{5e}$ or $\frac{32}{5}$	$e^{-1}$ . Allow 6.4e <sup>-1</sup> but not e.g. $\frac{64}{10e}$	A1
					[7]
		Q	uestion 10 l	Notes	7
	Note			ntiation e.g. $e^{5-2y} dx - 2xe^{5-2y} dy - dy =$	0

Note	The 2 <sup>nd</sup> and 3 <sup>rd</sup> method marks are available for work in decimals but the final method mark requires exact work.
Note	Accept $y'$ for $\frac{dy}{dx}$

Question Number	Scheme	Notes	Marks	
<b>11.</b> (a)	$x = 3\sec q = \frac{3}{\cos q} = 3(\cos q)^{-1}$			
	$\frac{\mathrm{d}x}{\mathrm{d}q} = -3(\cos q)^{-2}(-\sin q)$	$\frac{\mathrm{d}x}{\mathrm{d}q} = \pm k \Big( (\cos q)^{-2} (\sin q) \Big)$	M1	
	$\frac{dx}{dq} = \left\{ \frac{3\sin q}{\cos^2 q} \right\} = \left( \frac{3}{\cos q} \right) \left( \frac{\sin q}{\cos q} \right) = \underbrace{\frac{3\sec q \tan q}{\cos q}} *$ $\frac{dx}{d\theta} = \left\{ \frac{3\sin \theta}{\cos^2 \theta} \right\} = \underbrace{\left( \frac{3}{\cos \theta} \right) (\tan \theta)}_{Or} = \underbrace{\frac{3\sec \theta \tan \theta}{\sin \theta}}_{SCO} *$ $\frac{dx}{d\theta} = \left\{ \frac{3\sin \theta}{\cos^2 \theta} \right\} = \underbrace{\left( \frac{3\tan \theta}{\cos \theta} \right)}_{SCO} = \underbrace{\frac{3\sec \theta \tan \theta}{\cos \theta}}_{SCO} *$	Convincing proof with no notational or other errors such as missing $\theta$ 's or missing signs or inconsistent variables.  But use of $\cos^{-1}\theta$ as $\frac{1}{\cos\theta}$ is OK.  Must see both <u>underlined steps</u> .  Allow $3\tan\theta\sec\theta$	A1 *	
	If the $\frac{dx}{d\theta}$ is included on the lhs it must be correct but condone its omission and apply isw if possible if it appears correctly at some point in their working.			
			[2]	
(a) <b>Alt 1</b>	$x = 3\sec q = \frac{3}{\cos q}$			
	$x = 3\sec q = \frac{3}{\cos q}$ $\begin{cases} u = 3 & v = \cos q \\ \frac{du}{dq} = 0 & \frac{dv}{dq} = -\sin q \end{cases}$			
	$\frac{\mathrm{d}x}{\mathrm{d}q} = \frac{0(\cos q) - (3)(-\sin q)}{(\cos q)^2}$	Accept $\frac{0 \times (\cos \theta) \pm (3)(\sin \theta)}{(\cos \theta)^2}$ as evidence but if the quotient rule is quoted, it must be correct.	M1	
	$\frac{dx}{dq} = \left\{ \frac{3\sin q}{\cos^2 q} \right\} = \left( \frac{3}{\cos q} \right) \left( \frac{\sin q}{\cos q} \right) = \underbrace{\frac{3\sec q \tan q}{\cos q}} *$ $\frac{dx}{d\theta} = \left\{ \frac{3\sin \theta}{\cos^2 \theta} \right\} = \underbrace{\left( \frac{3}{\cos \theta} \right) \left( \tan \theta \right)} = \underbrace{\frac{3\sec \theta \tan \theta}{\cos^2 \theta}} *$	Convincing proof with no notational or other errors such as missing $\theta$ 's.  Must see both <u>underlined steps.</u> Allow $3\tan\theta\sec\theta$	A1 *	
	If the $\frac{dx}{d\theta}$ is included on the lhs it must be correct but			
	possible if it appears correctly at some	point in their working.	[2]	

(b)	$y = \frac{\sqrt{x^2 - 9}}{x}, x \ge 3; \ x = 3\sec\theta \Rightarrow \frac{dx}{d\theta} = 3\sec\theta\tan\theta$				
	$\int \frac{\sqrt{x^2 - x^2}}{x}$	$\frac{1}{1-9} dx = \int \frac{\sqrt{((3\sec\theta)^2 - 9)}}{3\sec\theta} 3\sec\theta \tan\theta d\theta$	Full substitution of $\frac{\sqrt{x^2-9}}{x}$ in terms of $q$ and "dx" as their " $\pm k \sec q \tan q$ ". This may be implied if they reach $\pm \lambda \int \tan^2 \theta \{d\theta\}$ with no incorrect working seen.	M1	
	Note	: If $\sqrt{x^2 - 9}$ is simplified incorrectly to $x - 3$ to substitution. (Any subsequent r	he first mark is still available for a full		
		_	$\pm \lambda \int \tan^2 \theta \{ d\theta \}$ (Allow $\pm \lambda \int \tan \theta \tan \theta \{ d\theta \}$ )	M1	
	$= 3 \int \tan^2 \theta  d\theta$ $= (3) \int (\sec^2 \theta - 1)  d\theta$		$3 \int \tan^2 \theta \{ d\theta \}$ (Allow $3 \int \tan \theta \tan \theta \{ d\theta \}$ )	A1	
			<b>Dependent on the previous M mark</b> applies $\tan^2 q = \sec^2 q - 1$	dM1	
	$= (3)(\tan \theta - \theta) \qquad k \tan^2 \theta \to k(\tan \theta - \theta)$			A1	
	$\left\{ \text{Area}(R) = \int_{3}^{6} \frac{\sqrt{(x^2 - 9)}}{x}  dx = \left[ 3\tan q - 3q \right]_{0}^{\frac{p}{3}} \right\}$				
		$= \left(3\tan\left(\frac{\rho}{3}\right) - 3\left(\frac{\rho}{3}\right)\right) - (0)$	Substitutes limits of $\frac{\rho}{3}$ and 0 into an expression that contains a trigonometric and an algebraic function and subtracts the correct way round. [ <b>Note:</b> Limit of 0 can be implied.] If they return to $x$ , they must substitute the limits 6 and 3 and subtract the correct way round having previously obtained a trigonometric and an algebraic function.	M1	
	$= 3\sqrt{3} - \rho$ $= 3\sqrt{3} - \rho$		A1		
	$[3\tan\theta - 3\theta]_0^{\frac{\pi}{3}} = 3\sqrt{3} - \pi \text{ can score the final M1A1 but if no substitution is shown and the are is incorrect, score M0}$				
		is medicet, score		[7]	
		Owastian 1	1 Notes	9	
<b>11.</b> (a)	Note $ x = \frac{3}{\cos \theta} \Rightarrow x \cos \theta = 3 \Rightarrow \frac{dx}{d\theta} \cos \theta - x \sin \theta = 0 \Rightarrow \frac{dx}{d\theta} = \frac{x \sin \theta}{\cos \theta} = 3 \sec \theta \tan \theta \text{ is M1A} $ $M1 \text{ for } \pm A \frac{dx}{d\theta} \cos \theta \pm B x \sin \theta = 0 $			A1.	
(b)	Note	A decimal answer of 2.054559769 (without			

Question Number	Scheme	Notes	Marks
12.	$\cot x - \tan x =$	$= 2\cot 2x$	
(a)	$\cot x - \tan x = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$	Attempts to write both $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$ only	M1
	$= \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\cos x \sin x} \left( = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \right)$	<b>Dependent on the previous M mark</b> Attempts to find the same denominator for both fractions	dM1
	$= \frac{\cos 2x}{\frac{1}{2}\sin 2x} \left( = \frac{2\cos 2x}{\sin 2x} \right)$	<b>Dependent on both the previous M</b> <b>marks.</b> Evidence of correctly applying either $\cos 2x = \cos^2 x - \sin^2 x$ or $\sin 2x = 2\sin x \cos x$	ddM1
	$= 2\cot 2x  (*)$	Correct proof with no notational or other errors such as missing <i>x</i> 's or inconsistent variables.	A1 *
			[4]
(a) <b>Alt 1</b>	$\cot x - \tan x = \frac{1}{\tan x} - \tan x$	Writes $\cot x$ in terms of $\tan x$	M1
	$\frac{1}{\tan x} - \frac{\tan^2 x}{\tan x} \left( = \frac{1 - \tan^2 x}{\tan x} \right)$	Dependent on the previous M mark Attempts to find the same denominator for both fractions	dM1
	$\frac{2}{\tan 2x}$	Dependent on both the previous M marks. Evidence of correctly applying $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	ddM1
	$= 2\cot 2x  (*)$	Correct proof with no notational or other errors such as missing <i>x</i> 's or inconsistent variables.	A1*
			[4]
(a) Alt 2	$2\cot 2x = \frac{2}{\tan 2x}$	Applies $\cot 2x = \frac{1}{\tan 2x}$	M1
	$=\frac{2}{\frac{2\tan x}{1-\tan^2 x}}$	Dependent on the previous M mark Attempts to apply the double angle formula for $\tan 2x$	dM1
	$= \frac{1 - \tan^2 x}{\tan x} = \frac{1}{\tan x} - \tan x$	Dependent on both the previous M marks. Obtains a rational fraction with a single denominator and attempts to split this up into 2 terms	ddM1
	$= \cot x - \tan x  (*)$	Correct proof with no notational or other errors such as missing <i>x</i> 's or inconsistent variables.	A1 *
			[4]

(b)	$5 + \cot(\theta - 15^{\circ}) - \tan(\theta - 15^{\circ}) = 0$				
		$\Rightarrow 5 + 2\cot() = 0$	Obtains an equation of this form.	M1	
		5 2	Obtains an equation of the form		
	C	$\cot(\dots) = -\frac{5}{2} \implies \tan(\dots) = -\frac{2}{5}$	$\tan() = \pm \frac{2}{5}$	M1	
	$2\theta - 30 = \tan^{-1}\left(-\frac{2}{5}\right)$		Can be implied by e.g.		
			$2\theta - 30 = \text{awrt} - 21.8$	A1	
			or	711	
			$2\theta - 30 = \text{awrt } 158.2$		
	θ	= awrt 4.1° <b>or</b> $\theta$ = awrt 94.1°	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1	
	$\theta =$	= awrt $4.1^{\circ}$ and $\theta$ = awrt $94.1^{\circ}$	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1	
					[5]
	Alternative to part (b):				
	$5 + \cot() - \tan() = 0 \Rightarrow 5\tan() + 1 - \tan^{2}()$ $\tan^{2}() - 5\tan() - 1 = 0$			M1	
	Multiples through by tan() to obtain a 3TQ in tan()			IVII	
	$\tan() = \frac{5 \pm \sqrt{25 + 4}}{2}$		Solves their 3TQ and proceeds to tan() =	M1	
	Can be implied by e.g.				
	$\theta = \text{awrt } 4.1^{\circ}$ or $\theta = \text{awrt } 94.1^{\circ}$ region $\theta = \text{awrt } 4.1^{\circ}$ and $\theta = \text{awrt } 94.1^{\circ}$ and $\theta = \text{awrt } 94.1^{\circ}$ and $\theta = \text{awrt } 94.1^{\circ}$		$\theta - 15 = 79.099$ or $\theta - 15 = -10.900$	A1	
			One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1	
			Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1	
					[5]
		Quart	ion 12 Notes	<u> </u>	9
			ates to "meet in the middle" e.g.		
			$\int x = \frac{1 - \tan^2 x}{\tan x}$ : M1dM1 as in Alt1		
(a)	Note		$\frac{2}{\frac{\ln x}{\ln^2 x}}$ : ddM1 uses double angle for tan2x on rhs		
		===	$\frac{1-\tan^2 x}{\cos^2 x}$ so lhs = rhs		
			tan x rect proof with conclusion		
		Al Con	tect proof with conclusion		

Question Number	Scher	me		Notes	Marks
<b>13.</b> (a)	$\frac{1}{(4-x)(2-x)} = \frac{1}{(4-x)(2-x)}$ $\Rightarrow 1 = A(2-x) + B(4-x)$	, , ,	$ \begin{array}{c c} 1 & A(2-x) \\ \hline  & 1 \\ \hline  & (4-x)(2-x) \end{array} $	orrect identity. For example, $+ B(4-x)$ from $\frac{A}{(4-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)}$ least one of $A =$ or $B =$	M1
	$A = -\frac{1}{2}, B = \frac{1}{2} \text{ giving}$		Cannot be remust be state and not just	or any equivalent form. ecovered from part (b) and ed as partial fractions in (a) the values of the constants.	A1
		Correct answer in (a)	scores both m	arks	[0]
(b)		$\frac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2$	$(-x), t \geqslant 0$		[2]
	$\int \frac{1}{(4-x)(2-x)}$	$dx = \int k  dt$	Separates variables correctly. $dx$ and $dt$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.		B1 oe
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2x)$	(x-x) = kt + c	$\pm \lambda \ln \alpha (4-x) \pm \mu \ln \beta (2-x),$ $\lambda \neq 0, \ \mu \neq 0, \ \alpha \neq 0, \ \beta \neq 0$		M1
	$ \begin{array}{cccc} 2 & 2 & \\ & \text{Or e.} \\ \frac{1}{2}\ln(8-2x) - \frac{1}{2}\ln(4x) & \\ \end{array} $	g.	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt \text{ oe}$ Do not condone missing brackets around the $4-x$ and/or the $2-x$ unless they are implied by subsequent work.		A1
	$\{t=0, x=0 \Rightarrow\} \frac{1}{2} \operatorname{lr}$		$c = \frac{1}{2}\ln 2$	Using both $t = 0$ and $x = 0$ in an integrated equation containing a constant of integration.	M1
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt + \frac{1}{2}\ln 2 \Rightarrow \ln\left(\frac{(4-x)}{2(2-x)}\right) = 2kt$				
		Starting from an equat	ion of the form		
	$\frac{4-x}{4-2x} = e^{2kt}$ a fully correct method		$(x-x) = \pm kt + c$ , $(x)$ , $(x)$ , $(x)$ , $(x)$ , $(x)$ , and applies to eliminate their logarithms. (Sign errors constant of integration that need not be		M1
	$4 - x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4 - 4e^{2kt} = x - 2xe^{2kt}$ $\Rightarrow 4 - 4e^{2kt} = x(1 - 2e^{2kt}) \Rightarrow x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}} $ (*)		A complete correct method of rearranging to make x the subject allowing sign errors only. Must have a constant of integration		dM1
		$1-2e^{2x}$	Achieves the	given answer with no errors.	A1 *
					[7]

(c)	{ -	$\frac{4-x}{4-2x} = e^{2kt}$ $\Rightarrow e^{2kt} = \frac{4-1}{4-2} \left\{ = \frac{3}{2} \right\}$	Substitutes $x = 1$ leading to $e^{2kt} = \text{value } \text{Note: } k = 0.1$	M1				
	$t = \frac{1}{2(0.1)}$	$-\ln\left(\frac{3}{2}\right) = 2.027325541 \left\{= 2.03 \text{ (s) (3 sf)}\right\}$	Anything that rounds to 2.03  Do not apply isw here and do not accept the exact value.	A1				
					[2]			
					11			
	Question 13 Notes							
	Note	May use an earlier form of their equation to find $t$ when $x = 1$ e.g.						
		$\frac{1}{2}\ln(3) - \frac{1}{2}\ln(1) = 0.1t + \frac{1}{2}\ln 2 \Rightarrow 0.2t = \ln\frac{3}{2}$						
		M1: For correct processing leading to $kt$ = value						
(c)		$t = \frac{1}{2(0.1)} \ln\left(\frac{3}{2}\right) = 2.027325541 \left\{= 2.03 \text{ (s) (3 sf)}\right\}$						
		A1: Anything that rounds to 2.03						
		Do not apply isw here						

Question Number	Scheme	Notes	Marks		
14.	(a) $y = \frac{(x^2 - 4)^{\frac{1}{2}}}{x^3}$ , $x > 2$ ; (b) $f(x) = \frac{24(x^2 - 4)^{\frac{1}{2}}}{x^3}$ , $x > 2$				
(a)	$u = (x^2 - 4)^{\frac{1}{2}} \qquad v = x^3$	$(x^2 - 4)^{\frac{1}{2}} \rightarrow \pm /x(x^2 - 4)^{-\frac{1}{2}}, \ \lambda \neq 0.$ Can be implied.	M1		
	$\frac{du}{dx} = \frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}}  \frac{dv}{dx} = 3x^2$	$(x^2 - 4)^{\frac{1}{2}} \rightarrow \frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}} \text{un-simplified}$ or simplified. <b>Can be implied.</b>	A1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}}(x^3) - 3x^2(x^2 - 4)^{\frac{1}{2}}}{(x^3)^2}$	Applies $\frac{vu\ell - uv\ell}{v^2}$ with $u = (x^2 - 4)^{\frac{1}{2}}$ , $v = x^3$ , their $u\ell$ and their $v\ell$ .	M1		
	$dx \qquad (x^3)^2$	Correct $\frac{dy}{dx}$ , un-simplified or simplified.	A1		
	$= \frac{x^4(x^2-4)^{-\frac{1}{2}}-3x^2(x^2-4)^{\frac{1}{2}}}{x^6}$				
	Either $ \frac{dy}{dx} = \frac{(x^2 - 4)^{-\frac{1}{2}}(x^4 - 3x^2(x^2 - 4))}{x^6} $	Simplifies $\frac{dy}{dx}$ by <b>either</b> correctly taking out a			
	$\frac{dy}{dx} = \frac{x^{6}}{x^{6}}$ or $\frac{dy}{dx} = \frac{x^{2}(x^{2} - 4)^{-\frac{1}{2}} - 3(x^{2} - 4)^{\frac{1}{2}}}{x^{4}}$	factor of $(x^2 - 4)^{-\frac{1}{2}}$ from their numerator <b>or</b> by multiplying numerator and denominator by $(x^2 - 4)^{\frac{1}{2}}$	M1		
	$\frac{dy}{dx} = \frac{x^2 - 3(x^2 - 4)}{x^4(x^2 - 4)^{\frac{1}{2}}} \Rightarrow \frac{dy}{dx} = \frac{-2x^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}}$	Correct algebra leading to $\frac{dy}{dx} = \frac{-2x^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}}$ $\left\{ A = -2 \right\}$	A1		
			[6]		
	Alternative by product rule: $u = (x^2 - 4)^{\frac{1}{2}}$ $v = x^{-3}$ $(x^2 - 4)^{\frac{1}{2}} \rightarrow \pm /x(x^2 - 4)^{-\frac{1}{2}}, \ \lambda \neq 0.$				
	u - (x - 4) $v - x$	Can be implied.	M1		
	$\frac{du}{dx} = \frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}}  \frac{dv}{dx} = -3x^{-4}$	$(x^2 - 4)^{\frac{1}{2}} \rightarrow \frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}}$ un-simplified or simplified. <b>Can be implied.</b>	A1		
	$\frac{dy}{dx} = \frac{1}{2}(2x)(x^2 - 4)^{-\frac{1}{2}}(x^{-3}) + (-3x^{-4})(x^2 - 4)^{\frac{1}{2}}$	Applies $vu' + uv'$ with $u = (x^2 - 4)^{\frac{1}{2}}$ , $v = x^{-3}$ , their $u^{\ell}$ and their $v^{\ell}$ .	M1		
	dx = 2	Correct $\frac{dy}{dx}$ , un-simplified or simplified.	A1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2(x^2 - 4)^{\frac{1}{2}}} - \frac{3(x^2 - 4)^{\frac{1}{2}}}{x^4} = \dots$	Simplifies $\frac{dy}{dx}$ by correctly writing as two fractions and attempts a common denominator	M1		
	$\frac{dy}{dx} = \frac{x^2 - 3(x^2 - 4)}{x^4(x^2 - 4)^{\frac{1}{2}}} \implies \frac{dy}{dx} = \frac{-2x^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}}$	Correct algebra leading to $\frac{dy}{dx} = \frac{-2x^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}}$ $\left\{ A = -2 \right\}$	A1		
		,	[6]		

r		T					
(b)	$ \left\{ f'(x) = \frac{24(-2x^2 + 12)}{x^4(x^2 - 4)^{\frac{1}{2}}} = 0 \Rightarrow \right\} $ $ 24(-2x^2 + 12) = 0 \Rightarrow x^2 = 6 $	Sets the numerator of their $\frac{dy}{dx} = 0$ or the numerator of their $f(x) = 0$ and solves to give $x^2 = K$ , where $K > 0$	M1				
	$\Rightarrow x = \sqrt{6} \text{ or awrt } 2.45$	$x = \sqrt{6}$ or awrt 2.45 (Allow $x = \pm \sqrt{6}$ or awrt $\pm 2.45$ ) (may be implied by their working)	A1				
	$f(\sqrt{6}) = \frac{24(6-4)^{\frac{1}{2}}}{(\sqrt{6})^3}; = \frac{24\sqrt{2}}{6\sqrt{6}} = \frac{4}{\sqrt{3}} \text{ or } \frac{4}{3}\sqrt{3}$	Dependent on the previous M mark. Substitutes their found $x$ into $f(x)$ or the given expression from part (a). May be implied by awrt 2.3 or may need to check their value.	dM1				
	$\left(\sqrt{6}\right)^3  ,  6\sqrt{6}  \sqrt{3}  3$	cso leading to $f_{max} = \frac{24\sqrt{2}}{6\sqrt{6}}$ or $\frac{4}{\sqrt{3}}$ or $\frac{4}{3}\sqrt{3}$ (Must be exact here)	A1				
	Range: $0 < f(x) \le \frac{4}{3}\sqrt{3}$ or $0 < y \le \frac{4}{\sqrt{3}}$ Or e.g. $\left(0, \frac{4}{3}\sqrt{3}\right]$	Correct range of $y$ or $f(x)$ . Also allow ft on their maximum <b>exact</b> value if both of the M's have been scored. Allow f or "range" for $f(x)$ .	A1ft				
			[5]				
(c)	The function f is many-one	Also accept "the function f is not one-one" or "the inverse is one-many".  This mark should be withheld if there are contradictory statements.	B1				
			[1]				
			12				
	Question 14 Notes						
14 (c)	Note  Accept  • f is many to one (or 2 values in domain of f map to one in the range)  • f is not one to one  • f -1 would be one to many  • it would be one to many  • it would be one to many  • it is not one to one  • the graph illustrates a many to one function  Do NOT allow  • it is many to one  • You can't reflect in y = x  Any reference to "it" we must assume refers to the inverse because of the wording in the question						
L	The state of the s						