Mark Scheme (Results)

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In Core Mathematics C34 (WMA02)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqcup$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Qu	Scheme	Marks
1	Differentiate wrt x $\quad \underline{3x^2} + \underline{\underline{6xy + 3x^2\frac{dy}{dx}}} + \underline{3y^2\frac{dy}{dx}} = \underline{0}$	M1 A1 B1
	Substitutes (1, 3) **AND** rearranges to get $\frac{dy}{dx}\left(=-\frac{7}{10}\right)$	M1
	$(y-3) = -\frac{7}{10}(x-1)$ so $7x + 10y - 37 = 0$	M1A1 **(6)**
		(6 marks)

M1 : Differentiates implicitly to include either $3x^2\frac{dy}{dx}$ **or** $3y^2\frac{dy}{dx}$ **term**

Accept $3x^2\frac{dy}{dx}$ appearing as $3x^2y'$ or $3y^2\frac{dy}{dx}$ as $3y^2y'$

A1: Differentiates $y^3 \to 3y^2\frac{dy}{dx}$ **and** $x^3 \to 3x^2$ **and** $37 \to 0$

B1: Uses the product rule to differentiate $3x^2y$ giving $6xy + 3x^2\frac{dy}{dx}$

Do not penalise students who write $3x^2dx + 6xydx + 3x^2dy + 3y^2dy = 0$

M1: Substitutes $x=1,\ y=3$ into their expression (correctly each at least once) to find a 'numerical' value for $\frac{dy}{dx}$ (may be incorrect). Note that $\frac{dy}{dx} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$

M1: Use of $(y-3) = m(x-1)$ where m is their numerical value of $\frac{dy}{dx}$

Alternatively uses $y = mx + c$ with $(1,3)$ and their m as far as c =..

A1: Accept integer multiples of the answer i.e. $7kx + 10ky - 37k = 0$ for example $21x + 30y - 111 = 0$

Note: If the gradient $-\frac{7}{10}$ just appears (from a graphical calculator) only M3 may be awarded

Qu	Scheme	Marks
2(a)	$\mathrm{f}(x) = x^3 - 5x + 16 = 0$ so $x^3 = 5x - 16$	M1
	$\Rightarrow x = \sqrt[3]{5x-16}$	A1
		(2)
(b)	$x_2 = \sqrt[3]{5 \times -3 - 16}$	M1
	$x_2 = -3.141$ awrt	A1
	$x_3 = -3.165$ awrt **and** $x_4 = -3.169$ awrt	A1
		(3)
(c)	$\mathrm{f}(-3.175) = -0.130984375..,\ \ \mathrm{f}(-3.165) = 0.120482875$ Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α lies in the interval $[-3.175, -3.165] \Rightarrow \alpha = -3.17$ (2 dp)	M1A1
		(2)
		(7 marks)

(a) Way 1:

M1: Must state f(x) = 0 (or imply by writing $x^3 - 5x + 16 = 0$) and reach $x^3 = \pm 5x \pm 16$

A1: completely correct with all lines including f(x) = 0 stated or implied (see above), $x^3 = 5x - 16$ and $x = \sqrt[3]{5x-16}$ oe with or without a = 5, b = −16. Isw after a correct answer

If a candidate writes $x^3 = 5x - 16 \Rightarrow x = (5x-16)^{1/3}$ then they can score 1 0 for a correct but incomplete solution.

Similarly if a candidate writes $x^3 - 5x + 16 = 0 \Rightarrow x = (5x-16)^{1/3}$

Way 2:

M1: starts with answer, cubes and reaches a =.., b = .

A**1**: Completely correct reaching equation and stating hence f(x) = 0

(b)

Ignore subscripts in this part, just mark as the first, second and third values given.

M1: An attempt to substitute $x_1 = -3$ into **their** iterative formula. E.g. Sight of $\sqrt[3]{-31}$, or can be implied by $x_2 = awrt - 3.14$

A1: $x_2 = \text{awrt} - 3.141$

A1: $x_3 = \text{awrt } -3.165$ **and** $x_4 = \text{awrt } -3.169$

(c)

M1: Choose suitable interval for x, e.g. $[-3.175, -3.165]$ and at least one attempt to evaluate f(x). Evidence would be the values embedded within an expression or one value correct. A minority of candidates may choose a tighter range which should include -3.1698 (alpha to 4dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Some candidates may use an adapted $\mathrm{f}(x) = 0$, for example $\mathrm{g}(x) = x - \sqrt[3]{(5x-16)}$ This is also acceptable even if it is called f, but you must see it defined. For your information $\mathrm{g}(-3.175) = -0.004, \mathrm{g}(-3.165) = (+)0.004$ If the candidate states an f (without defining it) it must be assumed to be $\mathrm{f}(x) = x^3 - 5x + 16$

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated)
(ii) sign change stated (>0, <0 acceptable as would a negative product) and
(iii) some form of conclusion which may be $\Rightarrow \alpha = -3.17$ or "so result shown" or qed or tick or equivalent

Qu	Scheme	Marks
3(a)	$\frac{9+11x}{(1-x)(3+2x)} = \frac{A}{1-x} + \frac{B}{3+2x}$ and attempt to find *A* or *B*	M1
	$A = 4$, $B = -3$	A1, A1
		(3)
(b)	$(1-x)^{-1} = 1 + x + x^2 + x^3 + ...$	B1
	$(3+2x)^{-1} = \frac{1}{3} \times \left(1 + (-1)\left(\frac{2}{3}x\right) + \frac{(-1)(-2)}{2}\left(\frac{2}{3}x\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{2}{3}x\right)^3 ...\right)$	B1 M1
	Attempts $'4' \times (.....) + '-3' \times (.....)$	M1
	$= 3 + \frac{14}{3}x + \frac{32}{9}x^2 + \frac{116}{27}x^3 ...$	A1, A1
		(6)
		(9 marks)

(a)
M1: For expression in markscheme or 9 + 11 x = *A* (3+2x) + *B* (1 – x) and use of substitution or comparison of coefficients in an attempt to find *A* or *B* (Condone slips on the terms)
A1: One correct value (this implies the M1)
A1: Both correct values (attached to the correct fraction).
You do not explicitly need to see the expression rewritten in PF form.
(b)
B1: Correct expansion $(1-x)^{-1} = 1 + x + x^2 + x^3 + ...$ with or without working. Must be simplified
B1: For taking out a factor of 3^{-1} Evidence would be seeing either 3^{-1} or $\frac{1}{3}$ before the bracket or could be implied by the candidate multiplying their *B* by $\frac{1}{3}$.
M1: For the form of the binomial expansion with $n = -1$ and a term of $\left(\pm\frac{2}{3}x\right)$.
To score M1 it is sufficient to see just any two terms of the expansion. eg. $1 + ... + \frac{(-1)(-2)}{2}\left(\pm\frac{2}{3}x\right)^2$
M1: Attempts to combine the two series expansions. Condone slips on signs but there must have been some attempt to combine terms (at least once) and to use both their coefficients
A1: Two terms correct which may be unsimplified.
A1: All four terms correct. (cao) Could be mixed number fraction form. ISW after a correct answer

Alternative use of binomial in line 2 of scheme: ie. $3^{-1} + (-1)(3)^{-2}(2x)$
B1: For seeing either 3^{-1} or $\frac{1}{3}$ as the first term
M1: It is sufficient to see just the first two terms (unsimplified) then marks as before

Way 2: Otherwise method: Use of $(9+11x)(1-x)^{-1}(3+2x)^{-1}$: B1 B1 M1 : as before
Then M1: Attempt to multiply three brackets and obtain 3 + .. A1: two terms correct A1: All four correct
Way 3: Use of $(9+11x)\left(3-\left(x+2x^2\right)\right)^{-1}$ or alternatives is less likely – send to review

Qu	Scheme	Marks
4.(a)	$0 < \mathrm{f}(x) < \frac{4}{5}$	M1A1 **(2)**
(b)	$y = \dfrac{4}{3x+5}$ $\Rightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \dfrac{4-5y}{3y}$	dM1
	$\mathrm{f}^{-1}(x) = \dfrac{4-5x}{3x}$ $\left(0 < x < \frac{4}{5}\right)$	A1o.e. **(3)**
(c)	$\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$	B1 (1)
(d)	$\dfrac{3x+5}{4} = \dfrac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4\times15\times15$ and so deduce no real roots	M1 A1 **(4)** **(10 marks)**

(a)
M1: One limit such as $y > 0$ or $y < 0.8$. Condone for this mark both limits but with x (not y) or with the boundary included. For example $[0, 0.8], 0 < x < 0.8, 0 \leqslant y \leqslant 0.8$

A1: Fully correct so accept $0 < \mathrm{f}(x) < \frac{4}{5}$ and exact equivalents $0 < y < \frac{4}{5}$ $(0, 0.8)$

(b)
M1: Set $y = \mathrm{f}(x)$ or $x = \mathrm{f}(y)$ and multiply both sides by denominator.
dM1:Make x (or a swapped y) the subject of the formula. Condone arithmetic slips

A1: o.e for example $y/\mathrm{f}^{-1}(x) = \frac{1}{3}\left(\frac{4}{x} - 5\right)$ or $y = \left(\frac{4}{x} - 5\right)\Big/3$ - do not need domain for this mark. ISW after a correct answer.

(c) Mark parts c and d together

B1: $\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$ - allow any correct form then isw

(d)
M1: Sets $\mathrm{fg}(x) = \mathrm{gf}(x)$ with **both sides correct** (but may be unsimplified) and forms a quadratic in x. Do not withhold this mark if fg or gf was originally correct but was "simplified" incorrectly and set equal to a correct gf
A1: Correct 3TQ. It need not be all on one side of the equation. The =0 can be implied by later work
M1: Attempts the discriminant or attempts the formula or attempts to complete the square.
A1: Completely correct work (cso) and conclusion. If $b^2 - 4ac$ has been found it must be correct (-576)

Qu	Scheme	Marks
5.(a)	$\frac{7\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{7\pi\sqrt{2}}{8}$ AND $\frac{9\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{9\pi\sqrt{2}}{8}$	B1 **(1)**
(b)	$\frac{1}{2}\times\frac{\pi}{4}\times\{.........................\}$	B1 oe
	$\frac{1}{2}\times h\times\left\{0+0+2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)\right\}$	M1
	= 11.91 (only)	A1 **(3)**
(c)	$\int x\cos x\,dx \quad = [x\sin x]-\int \sin x\,dx$	M1
	$= x\sin x + \cos x\ (+c)$	dM1 A1 **(3)**
(d)	$[x\sin x+\cos x]_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \quad = \frac{5\pi}{2}+\frac{3\pi}{2}=4\pi$	M1 A1 **(2)** **(9 marks)**

(a)
B1: Both correct (as above) Must be exact and not decimal
(b)
B1: See $\frac{1}{2}\times\frac{\pi}{4}\times$ as part of trapezium rule or $h=\frac{\pi}{4}$ stated or used. This can be scored if 'h' is in an unsimplified form.
M1: Correct structure of the bracket in the trapezium rule.
You may not see the zero's Eg $2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)$
A1: 11.91 only. This may be a result of using the decimal equivalents. Sight of 11.91 will score all 3 marks
(c)
M1: For a correct attempt at integration by parts to give an expression of the form $[\pm x\sin x]-\int\pm\sin x\,dx$
If you see such an expression you would only withhold the mark if there is evidence of an incorrect formula (seen or implied) Eg $\int udv = uv + \int vdu$
dM1: For $\pm\ x\sin x \pm \cos x$ following line one
A1: cso
Allow all three marks for candidate who just writes down the correct answer with no working

Watch for candidates who write down methods like this.

D	I
x	$\cos x$
1	$\sin x$
0	$-\cos x$

and then write $x\sin x-(-\cos x)$

This is a commonly taught algorithm (They differentiate down the lh column and integrate on the rh column. The answer is found by $D1\times I2-D2\times I3$ where $D2$ is the second entry in the D column. This can score full marks for the answer $x\sin x+\cos x$ but also pick up methods for slips.
If they attempt $D1\times I2+D2\times I3$ it is M0 as they are implying an incorrect formula. Ask your TL if unclear.
(d)
M1: Using limits $\frac{5\pi}{2}$ and $\frac{3\pi}{2}$ correctly in their answer to part (c) - substituting (seen correctly in all terms or implied in all terms) and subtracting either way around
A1: 4π or equivalent single term. CSO. It must have been derived from $x\sin x + \cos x$

Qu	Scheme	Marks
6.(i)	$\frac{dy}{dx} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1
		(2)
(ii)	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin x + \cos x)^2}$	M1
	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin^2 x + \cos^2 x) + (2\sin x \cos x)} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{1 + \sin 2x}$	B1, B1
	$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}$ *	A1 *
		(4)
		(6 marks)

(i)
M1: Applies the Product rule to $y = 5x^2 \ln 3x$

Expect $\frac{dy}{dx} = Ax + Bx\ln(3x)$ for this mark (A, B positive constant)

A1: cao- need not be simplified

(ii)
M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$

Expect $\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\pm\cos x \pm \sin x)}{(\sin x + \cos x)^2}$ for M1

Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator
Eg $\sin^2 x + \cos^2 x$

B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + ...$ and $(\sin^2 x + \cos^2 x) \to 1$

B1: Denominator should be expanded to $... + k\sin x \cos x$ and $(k \sin x \cos x) \to \frac{k}{2}\sin 2x$.

For example sight of $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x = 1 + \sin 2x$ without the intermediate line on the Denominator is B0 B1

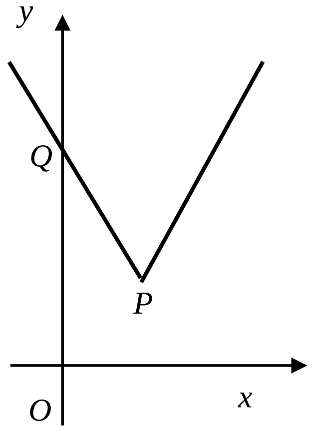
A1: cso – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos$ $\sin^2 x \leftrightarrow \sin x^2$
If the only error is the omission of $(\sin^2 x + \cos^2 x) \to 1$ then this final A1* can be awarded.

.
Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct

Product rule $\frac{dy}{dx} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2}(\pm\cos x \pm \sin x)$

Implicit differentiation $(\sin x + \cos x)y = x \Rightarrow (\sin x + \cos x)\frac{dy}{dx} + y(\pm\cos x \pm \sin x) = 1$

To score the B's under this method there must have been an attempt to write $\frac{dy}{dx}$ as a single fraction

Qu	Scheme	Marks
7 (a)(i)	Substitute (0, 5) to give $\lvert b \rvert = 5$ so $b = \pm 5$ Substitute $\left(\frac{1}{3}, 0\right)$ to give $\left\lvert \frac{1}{3}a + b \right\rvert = 0$ so $a = \mp 15$	B1 M1 A1
(ii)	Gives equation as $y = \lvert -15x + 5 \rvert$ or $y = \lvert 15x - 5 \rvert$	B1 **(4)**
(b)	 V shape correct way up P at $\left(\frac{2}{3}, 3\right)$ Q at (0, 8)	B1 B1 B1 **(3)** **(7 marks)**

(a)
(i)
B1: For both $b = \pm 5$ not just $\lvert b \rvert = 5$
M1: Substitute $\left(\frac{1}{3}, 0\right)$ to give $\left\lvert \frac{1}{3}a + b \right\rvert = 0$. This mark is implied by $a = \pm 3 \times$ value of b
A1: $a = 15$ corresponding to $b = -5$ **and** $a = -15$ corresponding to $b = 5$

If they write down an equation rather than giving values of a and b then
Just $y = \lvert -15x + 5 \rvert$ **or** $y = \lvert 15x - 5 \rvert$ scores B0, M1, A0
Both $y = \lvert -15x + 5 \rvert$ **and** $y = \lvert 15x - 5 \rvert$ scores B1, M1, A1
Linear equations $y = -15x + 5$ and/or $y = 15x - 5$ (without the modulus) only score B0 M1 A0

(ii) Note that this is an A1 mark on e-pen

B1: $y = \lvert -15x + 5 \rvert$ or $y = \lvert 15x - 5 \rvert$ or allow equations such as for this mark only $\mathrm{f}(x) = \begin{cases} 15x - 5 & x \geqslant \frac{1}{3} \\ -15x + 5 & x < \frac{1}{3} \end{cases}$

If candidates don't state (i), (ii) and write down just $y = \lvert -15x + 5 \rvert$ they would score (i) B0 M1 A0 (ii) B1
(b) There must be a sketch to score any of these marks.
B1: V shape the correct way up any position but not on the x- axis. Accept V's that don't have symmetry
B1: P at $\left(\frac{2}{3}, 3\right)$ Score if the coordinates are stated within the text OR marked on the axes. If they appear in both then the graph takes precedence.
B1: For **crossing** the y- axis at (0, 8). Accept 8 marked on the correct axis. Condone (8,0) marked on the correct axis

Qu	Scheme	Marks
8 (a)	$\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$	M1
	$=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ OR $=\dfrac{\dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x}}{\dfrac{1-\tan^2 x-2\tan^2 x}{1-\tan^2 x}}$ oe	A1
	So $\tan 3x=\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*cso
		(4)
(b)	Put $\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}=11\tan x$ so $3\tan x-\tan^3 x=11\tan x(1-3\tan^2 x)$	M1
	$32\tan^3 x=8\tan x$	A1
	So $\tan x = \pm\dfrac{1}{2}$ or $0 \Rightarrow x=..$	dM1
	$\Rightarrow x=$ awrt $26.6°$, $-26.6°$, 0	A1 A1
		(5)
		(9 marks)

(a)

M1: Expands $\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ condoning sign errors

dM1: Uses the correct double angle formula $\dfrac{2\tan x}{1-\tan^2 x}$ both times within their expression for tan(2*x*+*x*)

A1: Multiplies both numerator and denominator by $1-\tan^2 x$ to obtain a correct intermediate line

Eg $=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ or similar.

Alternatively they write both numerator and denominator as single correct fractions.
They cannot just write down the final given answer for this mark

A1*: Correct printed answer achieved with no errors and all of the lines in the markscheme (c.s.o.)
Withhold the final A1 for candidates who use poor notation or mixed variables.

Examples of poor notation would include $\tan \leftrightarrow \tan x$ $\tan^2 x \leftrightarrow \tan x^2$ $\tan 2x=\dfrac{2\tan\theta}{1-\tan^2\theta}$

(b)

M1: Attempts to use the given identity and multiplies by $1-3\tan^2 x$. Condone slips

A1: Obtain $32\tan^3 x=8\tan x$ or equivalent. Accept $32\tan^2 x=8$ for this mark

dM1: Obtains one value of *x* from tan *x* =... using a correct method for their equation. The order of operations to find *x* must be correct but can be scored from $\tan x=0\Rightarrow x=0$

A1: Either one of $x=26.6°$ or $-26.6°$ or in radians ± 0.46

A1: CAO $x=$ awrt $26.6°$, awrt $-26.6°$, 0 (do not need degrees symbol) with no extras within the range

Note: Answers only scores 0 marks. Answers from a correct cubic/quadratic scores M1 A1 dM1 (implied) then as scheme

Qu	Scheme	Marks
9(a)	$\frac{du}{dx}=2$	B1
	$\left(\int \frac{x}{(2x+3)^2}\,dx\right)=\int \frac{u-3}{4u^2}du$	M1
	$=\int \frac{1}{4}u^{-1}-\frac{3}{4}u^{-2}\,du$	dM1
	$=\frac{1}{4}\ln u+\frac{3}{4}u^{-1}$	ddM1 A1
	$\left[\frac{1}{4}\ln u+\frac{3}{4}u^{-1}\right]_3^{27}=\frac{1}{4}\ln 27+\frac{3}{4}\times\frac{1}{27}-\left(\frac{1}{4}\ln 3+\frac{3}{4}\times\frac{1}{3}\right)=\frac{1}{4}\ln 9-\frac{8}{36}$	M1
	$=\frac{1}{2}\ln 3-\frac{2}{9}$ *	A1*
		(7)
(b)	$V=\pi\times\int_0^{12}\left(\frac{9\sqrt{x}}{2x+3}\right)^2dx$	M1
	$=81\pi\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$	A1
		(2)
		(9 marks)

(a)

B1: States or uses $\frac{du}{dx}=2$ or equivalent such as $\frac{dx}{du}=0.5$ or $dx=\frac{1}{2}du$

M1: Expression of the form $k\int \frac{u-3}{u^2}du$ and allow missing du and/or missing integral sign

d M1: Splits into the form $...u^{-1}\pm...u^{-2}$ and again allow missing du and/or missing integral sign.

Alternatively they could use integration by parts at this stage $\int \frac{u-3}{4u^2}du=\pm a\frac{(u-3)}{u}\pm b\int\frac{1}{u}du$ A small number of candidates will also use partial fractions that gives the same answer as the main scheme.

ddM1: For $...\ln u\pm...u^{-1}$ or 'obviously $...\ln 4u\pm...u^{-1}$ or by parts $\pm a\frac{(u-3)}{u}\pm b\ln u$

A1: $\frac{1}{4}\ln u+\frac{3}{4}u^{-1}$ This answer or equivalent such as $\frac{1}{4}\ln 4u+\frac{3}{4}u^{-1}$ or by parts $-\frac{(u-3)}{4u}+\frac{1}{4}\ln u$

M1: Applies limits of 27 and 3 to the result of integrating their function in u, subtracts the correct way around and combines the ln terms correctly. Alternatively using $u=2x+3$ applies the limits x = 12 and 0 to the result of their adapted function subtracts the correct way round and combines the ln terms correctly.

A1*: given answer achieved correctly without errors. The only omission that would be allowed could be the dM1 line which could be implied. You need to see an intermediate step with correct ln work before the final answer is reached.

(b)

M1: Attempts to use part (a) to find the exact volume. Accept $\pi\times\int_0^{12}\left(\frac{9\sqrt{x}}{2x+3}\right)^2dx$

Condone only the omission of π or 81 or a bracket for this M mark so accept $81\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$ or $\pi\left(\frac{1}{2}\ln 3-\frac{2}{9}\right)$ or $\frac{1}{2}\ln 3-\frac{2}{9}\times 81\pi$ as evidence

A1: Any correct exact equivalent in terms of ln3 and π Accept for example $81\pi\left(\ln\sqrt{3}\right)-18\pi$

A correct answer implies both marks. Remember to isw after a correct answer.

It is possible to do 9(a) by parts or via partial fractions **without** using the given substitution. This does not satisfy the demands of the question but should score some marks. A fully correct solution via either method scores 5 out of 7

Qu	Scheme		Marks
9(a)	By Parts	By Partial Fractions	B0
		$\int x(2x+3)^{-2}\,dx = \int \frac{\frac{1}{2}}{(2x+3)} + \frac{-\frac{3}{2}}{(2x+3)^2}\,dx$	
	$\int x(2x+3)^{-2}\,dx = \frac{x(2x+3)^{-1}}{-2} + \int \frac{(2x+3)^{-1}}{2}\,dx$	One term of $= \frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	M1
	$= \frac{x(2x+3)^{-1}}{-2} + \frac{1}{4}\ln(2x+3)$	Both terms of $= \frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}$	dM1
	Attempts limits $= \left[\frac{x(2x+3)^{-1}}{-2} + \frac{1}{4}\ln(2x+3)\right]_0^{12}$	$= \left[\frac{1}{4}\ln(2x+3) + \frac{3(2x+3)^{-1}}{4}\right]_0^{12}$	ddM1
	Correct un simplified answer $= -\frac{2}{9} + \frac{1}{4}\ln 27 - \frac{1}{4}\ln 3$	$= \frac{1}{4}\ln 27 + \frac{1}{36} - \frac{1}{4}\ln 3 - \frac{1}{4}$	A1
	Collects log terms $= -\frac{2}{9} + \frac{1}{4}\ln\left(\frac{27}{3}\right)$	$= -\frac{2}{9} + \frac{1}{4}\ln\left(\frac{27}{3}\right)$	M1
	$= \frac{1}{2}\ln 3 - \frac{2}{9}$		A0*
			(7)

Qu	Scheme	Marks
10.(a)	When $t = 0$ $N = 15$	B1 **(1)**
(b)	Puts $t = 10$ so $N =$ 56.6 (accept 56 or 57)	M1A1 **(2)**
(c)	$82 = \frac{300}{3+17e^{-0.2t}} \Rightarrow e^{-0.2t} = \frac{54}{1394} =$ awrt 0.039	M1 A1
	$-0.2t = \ln\left(\frac{54}{1394}\right) \Rightarrow t =$	dM1
	$t =$ awrt 16.3	A1 **(4)**
(d)	$\frac{dN}{dt} = (-0.2)\times 300\times(-1)\times 17e^{-0.2t}(3+17e^{-0.2t})^{-2}$	M1 A1
	=4.38 so 4 insects per week	A1 cso **(3)**
		(10 marks)

(a)
B1: 15 cao
(b)
M1: Substitutes $t = 10$ into the correct formula. Sight of $N = \frac{300}{3+17e^{-0.2\times 10}}$ is fine
A1: Accept 56 or 57 or awrt 56.6. These values would imply the M.
(c)
M1: Substitutes 82 and proceeds to obtain $e^{\pm 0.2t} = C$ Condone slips on the power
A1: For $e^{-0.2t} = \frac{27}{697}$ oe $e^{0.2t} = \frac{697}{27}$ oe Accept decimals Eg $e^{-0.2t} =$ awrt 0.039 or $e^{0.2t} =$ awrt 25.8
dM1: Dependent upon previous M, scored for taking ln's (of a positive value) and proceeding to $t =$
A1: awrt 16.3 Accept 16 (weeks), 16.25 (weeks), 16 weeks 2 days or 17 weeks following correct log work and acceptable accuracy. Accept $t = 5\ln\left(\frac{1394}{54}\right)$ *oe* for this mark

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It is possible to answer this by taking ln's at the point $1394e^{-0.2t} = 54$
M1A1 $\ln(1394) - 0.2t = \ln(54)$ dM1 A1 As scheme

(d)
M1: Differentiates to give a form equivalent to $\frac{dN}{dt} = ke^{-0.2t}(3+17e^{-0.2t})^{-2}$ (may use quotient rule)
A1: Correct derivative which may be unsimplified $\frac{dN}{dt} = 1020e^{-0.2t}(3+17e^{-0.2t})^{-2}$
A1: Obtains awrt 4 **following a correct derivative**. This is cso

Qu	Scheme	Marks
11. (a)	$R = 37$	B1
	$\tan\alpha = \frac{12}{35} \Rightarrow \alpha = \text{awrt } 0.3303$	M1 A1
		(3)
(b)	$\sin(x-\alpha) = \frac{37}{2R} \quad (= 0.5...)$	M1
	$x = \arcsin\left(\frac{37}{2\times\text{their "37"}}\right) + \text{their"0.3303"}$	M1
	$x = \text{awrt } 0.854$ or awrt 2.95	A1
	$x = \text{awrt } 0.854$ and awrt 2.95	A1
		(4)
(c)(i)	Find $y = \frac{7000}{31+(\pm\text{"}R\text{"})^2} = 5$	M1 A1
(c)(ii)	$x-\alpha = \frac{\pi}{2} \Rightarrow x = 1.90$	M1 A1
		(4)
		(11 marks)

(a)

B1: $R = 37$ no working needed. Condone $R = \pm 37$

M1: $\tan\alpha = \pm\frac{12}{35}$ or $\tan\alpha = \pm\frac{35}{12}$ with an attempt to find alpha. Accept decimal attempts from $\tan\alpha = \text{awrt } \pm 0.343$ or $\tan\alpha = \text{awrt } \pm 2.92$ If R is used allow $\sin\alpha = \pm\frac{12}{R}$ OR $\cos\alpha = \pm\frac{35}{R}$ with an attempt to find alpha

A1: $\alpha = \text{awrt } 0.3303$. Answers in degrees are A0

(b)

M1: (Uses part (a) to solve equation) $\sin(x\pm\alpha) = \frac{37}{2\times theirR}$

M1: operations undone in the correct order to give x =... Accept $\sin(x\pm\alpha) = k \Rightarrow x = \arcsin k \pm \alpha$

A1: one correct answer to within required accuracy. Allow 0.272π or 0.938π.

Condone for this mark only **both** $\frac{\pi}{6} + 0.3303$ **and** $\frac{5\pi}{6} + 0.3303$

A1: both values (and no extra values in the range) correct to within required accuracy. Allow $0.272\pi, 0.938\pi$

(c) (i)

M1: For an attempt at $\frac{7000}{31+(\pm\text{"}R\text{"})^2}$

A1: 5

(c)(ii)

M1: Uses $x - \text{their } \alpha = (2n+1)\frac{\pi}{2}$ to find x This may be implied by $1.57 \pm \text{their } 0.33$ stated or calculated (2dp)

A1: Awrt 1.90 but condone 1.9 for this answer

Answers in degrees, withhold the first time seen, usually part (a). FYI (a) $18.92°$ (b) $48.9°, 168.9°$ (c)(ii) $108.9°$

Qu	Scheme		Marks
	Way 1:	Way 2:	
12. (a)	Uses $x = kt$ or $t = cx$ and $x = 1.5$ when $t = 2$ so $k =$ or $c =$	Uses $x = kt + c$ with $x = 0$, $t = 0$ and with $x = 1.5$ when $t = 2$ so $k =$	M1
	$t = \frac{4}{3}x$	$t = \frac{4}{3}x$	A1 **(2)**
(b)	$t = 4$		B1 **(1)**
(c)	$\frac{dx}{dt} = \frac{\lambda}{(2x+1)}$ so separate variables to give $\int (2x+1)\,dx = \int \lambda\,dt$		M1
	$x^2 + x = \lambda t (+c')$ or $\frac{(2x+1)^2}{4} = \lambda t (+c)$ so $t =$		M1
	(When $t = 0$, $x = 0$ so $c = ¼$ or $c' = 0$) so $t = \frac{x^2 + x}{\lambda}$		A1 **(3)**
(d)	Uses $x = 1.5$ when $t = 2$ to give $\lambda = \frac{15}{8}$		B1 **(1)**
(e)	$t = \frac{x^2+x}{\lambda} = \frac{12}{\lambda}$ = 6.4 hours later so		M1
	10.24pm or 22.24		A1 **(2)** **(9 marks)**

Mark (a) and (b) together

(a)

M1: Uses correct $x = kt$ or $t = cx$ *and* $x = 1.5$ when $t = 2$ to find their constant (may not be *k or c*)

This may be the result of a differential equation $dx/dt = k$

A1: $t = \frac{4}{3}x$ oe such as $t = \frac{x}{0.75}$ or even $t = \frac{x}{3/4}$ Just this with no working is M1 A1

(b)

B1: $t = 4$

Mark (c),(d) and (e) together

(c)

M1: Correct separation but condone missing integral signs

M1: Correct form for both integrals- may not find *c* or even include a *c*

A1: Obtains a correct answer for *t* in terms of *x* and λ by using either $x = 0, t = 0 \Rightarrow t = \frac{x^2 + x}{\lambda}$ or

$t = \frac{(2x+1)^2 - 1}{4\lambda}$ oe. Alternatively uses $x = 1.5, t = 2 \Rightarrow t = \frac{4x^2 + 4x + 8\lambda - 15}{4\lambda}$ oe

Condone correct responses where '*c*' seems to have been either cancelled out or ignored

(d)

B1: $\lambda = \frac{15}{8}$ or decimal i.e. 1.875

(e)

M1: Substitutes $x = 3$ into their expression for t. Implied by $t = \frac{12}{"\lambda"}$

A1: 10.24pm or 22:24 only

Qu	Scheme	Marks
13 (a)	Puts $x = 0$ and obtains $\theta = -\frac{\pi}{6}$ Substitutes their θ to obtain $y = \frac{10\sqrt{3}}{3}$ or $\left(0, \frac{10\sqrt{3}}{3}\right)$	B1 M1 A1 **(3)**
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{5\sec\theta\tan\theta}{\sqrt{3}\sec^2\theta}$ $= \frac{5\times \sin\theta/\cos\theta}{\sqrt{3}\times 1/\cos\theta}$ $= \frac{5}{\sqrt{3}}\sin\theta$ or $\lambda = \frac{5}{\sqrt{3}}$ oe	M1 A1 B1 A1 **(4)**
(c)	Puts $\frac{dy}{dx} = 0$ and obtains θ and calculates x and y **or** deduces correct answer Obtains (1, 5)	M1 A1 **(2)**
(d)	$\tan\theta = \frac{x-1}{\sqrt{3}}$ and $\sec\theta = \frac{y}{5}$ Uses $1+\tan^2\theta = \sec^2\theta$ to give $1+"\left(\frac{x-1}{\sqrt{3}}\right)^2" = "\left(\frac{y}{5}\right)^2"$ $\frac{3+x^2-2x+1}{3} = \left(\frac{y}{5}\right)^2$ **so** $y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	M1 M1 A1* **(3)** **(12 marks)**
Alt 1(d)	$y = 5\sqrt{1+\tan^2\theta}, = 5\sqrt{1+\left(\frac{x-1}{\sqrt{3}}\right)^2}$ $y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	M1, M1 A1* (3)
Alt 2 (d)	Assume $y = k\sqrt{(x^2-2x+4)}$ and sub both $x = 1+\sqrt{3}\tan\theta$ and $y = 5\sec\theta$ $5\sec\theta = k\times\sqrt{3+3\tan^2\theta}$ $5\sec\theta = k\times\sec\theta\sqrt{3}$ $k = \frac{5}{3}\sqrt{3}$ AND conclusion "hence true"	M1 M1 A1* (3)

(a)

B1: For $\theta = -\frac{\pi}{6}$ or $-30°$ or awrt -0.52 but may be awarded for $\cos\theta = \frac{\sqrt{3}}{2}$ or $\sec\theta = \frac{2}{\sqrt{3}}$ if θ is not explicitly found

M1: Substitutes their θ (or their $\cos\theta$ or $\sec\theta$) found from an attempt at $x = 0$ to give y

A1: cao. Accept $y = \frac{10}{\sqrt{3}}$ Correct answer with no incorrect working scores all 3 marks.

Note that $\theta = \frac{\pi}{6}$ also gives $y = \frac{10\sqrt{3}}{3}$ but scores B0 M1 A0

Qu	Scheme	Marks
	(b) **M1:** Attempts to differentiate both x and y wrt θ and establishes $\left(\frac{dy}{dx}\right)=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ **A1:** Correct derivatives and correct fraction **B1:** For either $\lambda=\frac{5}{3}\sqrt{3}$ (seen explicitly stated or implied) or use of $\sec\theta=\frac{1}{\cos\theta}$ An alternative to seeing $\sec\theta=\frac{1}{\cos\theta}$ is $\frac{1}{\sec\theta}=\cos\theta$ **A1:** Fully correct solution showing all relevant steps with correct notation, no mixed variables and no errors. $\frac{\tan\theta}{\sec\theta}$ cannot just be written as $\sin\theta$ without an intermediate line of working $\frac{(\cos\theta)^{-2}\sin\theta}{\sec^2\theta}$ cannot just be written as $\sin\theta$ without an intermediate line of working However it is acceptable to write down $\tan\theta\cos\theta$ as $\sin\theta$ due to this being a version of $\frac{\sin\theta}{\cos\theta}=\tan\theta$ (c) **M1:** Sets their $\frac{dy}{dx}=0$ and proceeds to find (x, y) from their θ **A1:** for (1, 5) or x = 1, y = 5 (d) **M1:** Attempt to obtain $\tan\theta$ and $\sec\theta$ in terms of x and y respectively. Allow $\tan\theta=\frac{x\pm1}{\sqrt{3}}$ $\sec\theta=\frac{y}{5}$ **M1:** Uses $1+\tan^2\theta=\sec^2\theta$ with their expressions for $\tan\theta$ and $\sec\theta$ in terms of x and y respectively **A1*:** Obtains printed answer with no errors and with $k=\frac{5}{3}\sqrt{3}$ only You do not need to see k explicitly stated as $\frac{5}{3}\sqrt{3}$, it is fine to be embedded within the formula	

Qu	Scheme	Marks
14 (a)	Attempts $\overrightarrow{BA}=\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ or $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$ either way around	M1
	Finds $\overrightarrow{OD}\ =\mathbf{a}-\mathbf{b}+\mathbf{c}=(-2\mathbf{i}+2\mathbf{j}-8\mathbf{k})+(-\mathbf{i}+3\mathbf{j}+6\mathbf{k})=-3\mathbf{i}+5\mathbf{j}-2\mathbf{k}$	M1 A1
		(3)
(b)	$\overrightarrow{BA}\ =\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ and $\overrightarrow{BC}\ =\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$	M1
	$\cos\theta=\dfrac{\begin{pmatrix}-2\\2\\-8\end{pmatrix}.\begin{pmatrix}-4\\4\\0\end{pmatrix}}{\sqrt{(-2)^2+2^2+(-8)^2}\sqrt{(-4)^2+4^2}}=\dfrac{16}{\sqrt{72}\sqrt{32}}=\dfrac{1}{3}$	dM1 A1
	So angle is 1.23 radians or 70.5 degrees	A1
		(4)
(c)	Area = $\sqrt{72}\sqrt{32}\sin\theta=45.3$ **or** $32\sqrt{2}$ **oe**	M1A1
		(2)
(d)	Area = $\dfrac{3}{2}\times\text{"}45.3\text{"}=67.9$ or $48\sqrt{2}$ **oe**	M1 A1
		(2)
		(11 marks)

(a)
M1: For attempting one of **b – a** or **a – b or c – b** or **b – c.** It must be correct for at least one of the components. Condone coordinate notation for the first two M marks
M1: For attempting $\mathbf{d}=\mathbf{a}-\mathbf{b}+\mathbf{c}$ = It must be correct for at least one of the components.
A1: cao. Correct answer no working scores all 3 marks. It must be the vector (either form) and not a coordinate
Note this can be attempted by finding the mid point *E* of *AC* and then using **d** =**b**+2$\overrightarrow{BE}$ but it must be a full method M1 Attempts $mp_{AC}=(0,2,2)$ **and uses** .... M1 Attempts $(3,-1,6)+2\times(-3,3,-4)$ A1
(b)
M1: Uses correct pair of vectors , so $\pm k\overrightarrow{BA}$ and $\pm k\overrightarrow{BC}$. Each must be correct for at least one of the components
dM1: A clear attempt to use the dot product formula to find $\cos\theta=k, -1<k<1$. It is dependent upon having chosen the correct pair of vectors. Allow for arithmetical slips in both their dot product calculation and the moduli, but the process must be correct.
It could also be found using the cosine rule. $\dfrac{72+32-72}{2\sqrt{72}\sqrt{32}}=$
(M1 is for attempt at all three lengths, so $\pm\overrightarrow{BA}, \pm\overrightarrow{BC}, \pm\overrightarrow{AC}$ and dM1 correct angle attempted using the correct formula)
A1: For 1/3 or – 1/3 or equivalent - may be implied by 70.5 or 109.5 or 1.23 radians or 1.91 radians
A1: cso for awrt 70.5 degrees or 1.23 radians. (Note that invcos(– 1/3)=109.5 followed by 70.5 is A0 .... unless accompanied by a convincing argument that the angle 109.5 is the exterior angle, and therefore the interior angle is 70.5. It is not awarded for simply finding the acute angle. A diagram with correct angles would be ok)
(c)
M1: Uses correct area formula for parallelogram.
You may see the area of the triangle *ABC* doubled which is fine.
A1: Obtains awrt 45.3. Allow this from an angle of 109.5
(d)
M1: Realises connection with part (c) and uses 1.5 times answer to the area of *ABCD* (It can be implied by 67.9)
A1: awrt 67.9

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