



Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced
Level
In Mechanics M3 (WM03)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
 - the symbol \checkmark will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working

- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- * means the answer is printed on the question paper
- □ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
 - correct no. of terms;
 - dimensionally correct;
 - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

M(A)	Taking moments about A.
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

Question Number	Scheme	Marks
1.	$\int_0^3 \sqrt{x+1} dx$	M1
	$= \frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3$	A1
	$\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$ or $\frac{\int_0^3 \frac{1}{2} (x+1) dx}{\int_0^3 \sqrt{x+1} dx}$	M1
	$= \frac{\frac{1}{2} \left[\frac{1}{2} x^2 + x \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$ or $\frac{\frac{1}{2} \left[\frac{1}{2} (x+1)^2 \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$	A1
	$= \frac{45}{56}$ (0.80 or better)	A1 (5)
		(5)
Notes		
M1	Use of $\int_0^3 \sqrt{x+1} dx$. Limits not needed. Accept $k \times \int_0^3 \sqrt{x+1} dx$ where k is a constant. To give the mark for 'use' we must see an attempt at integration. An attempt at integration is seen when the powers increase by 1.	
A1	Correct integrated expression with correct limits	
M1	Use of $\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$. Limits not needed. The formula must be correct but allow a constant multiple if it appears on both numerator and denominator. We must see the correct formula and an attempt at integrating the numerator	
A1	Correct integrated expression for the numerator in the correct formula with correct limits.	
A1	Correct answer. This question comes with a calculator warning: the correct answer must come from integrated expressions ie both previous A's must have been awarded. Numerical substitution does not need to be seen.	

Question Number	Scheme	Marks
2.	$T = mg \cos \theta$	M1A1
	$T = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	M1A1
	$\frac{4}{5}mg = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	dM1
	$k = \frac{3}{2}$ or 1.5	A1
		(6)
	Notes	
M1	Resolve parallel to the string, correct no. of terms, condone sign errors and sin/cos confusion (or resolve in two directions and eliminate the unknown force or use trig on a right-angled triangle of forces) to give an equation in T , mg and θ only	
A1	Correct equation. Trig does not need to be substituted.	
M1	Use Hooke's Law with correct structure.	
A1	Correct equation	
dM1	Substitute trig and eliminate T to produce equation in k only, dependent on previous M's. If x is used for extension, should see $x = \frac{2k}{5}a \rightarrow ka + \frac{2k}{5}a = \frac{21}{10}a$	
A1	cao	
ALT 1	First M1A1	
M1A1	Complete method to form an equation in T and θ Vert: $mg = T \cos \theta + F \sin \theta$ Horiz: $F \cos \theta = T \sin \theta$ Eliminate F eg $\frac{\sin \theta}{\cos \theta} = \frac{mg - T \cos \theta}{T \sin \theta}$	

Question Number	Scheme			Marks	
3(a)	Mass:	$\frac{1}{3}\pi r^2 H$	$\frac{1}{3}\pi r^2 h$	$\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$	B1
	Distance from V:	$\frac{3H}{4}$	$H - \frac{1}{4}h$	\bar{x}	B1
	$\frac{1}{3}\pi r^2 H \times \frac{3H}{4} - \frac{1}{3}\pi r^2 h \times \left(H - \frac{1}{4}h\right) = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right) \bar{x}$			M1A1	
	$\bar{x} = \frac{(3H-h)(H-h)}{4(H-h)} = \frac{1}{4}(3H-h) *$			A1*	
				(5)	
3(b)					
	$M(G), T_1 \bar{x} = T_2 (H - \bar{x})$			M1	
	$\frac{T_1}{T_2} = \frac{H - \frac{1}{4}(3H-h)}{\frac{1}{4}(3H-h)}$			A1	
	$\frac{T_1}{T_2} = \frac{H+h}{3H-h}$			A1 (3)	
				(8)	
Notes					
3(a)					
B1	Three correct mass ratios: H h $H-h$				
B1	Three correct distances (Allow if measured from some other axis)				
M1	Moments equation with correct no. of terms, dim correct. Condone addition, treat as a sign error. Must be working with solids eg not a conical shell.				
A1	Correct unsimplified equation (For their axis)				
A1*	Given answer correctly obtained, including cancelling $(H-h)$. A factorised expression should be seen to reach the GIVEN answer.				
3(b)					
M1	Complete method to obtain an equation in H, h, T_1 and T_2 only. Must be dimensionally correct. May take moments about G and substitute for \bar{x} . Alternatively may use two equations, eliminate weight and substitute for \bar{x} .				
A1	Correct equation in T_1, T_2, H and h only				
A1	Correct answer. The question asks for simplest form.				
3(a) ALT 1	Distances measured from circular face				
B1	Mass:	$\frac{1}{3}\pi r^2 H$	$\frac{1}{3}\pi r^2 h$	$\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$	
B1	Dist from Circular face:	$\frac{H}{4}$	$\frac{1}{4}h$	d	
M1A1	$\frac{1}{3}\pi r^2 H \times \frac{H}{4} - \frac{1}{3}\pi r^2 h \times \frac{1}{4}h = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right) d$				

Question Number	Scheme	Marks
A1*	Leads to $d = \frac{H+h}{4}$ which must be subtracted from H to reach the required distance: $\bar{x} = H - \frac{H+h}{4} = \frac{1}{4}(3H-h) *$	
3(b) ALT 1		
M1	May use two equations and eliminate weight/mass ratio to find an equation in terms of T_1 , T_2 , H and h only. Vert $T_1 + T_2 = W$ M(V), $W\bar{x} = T_2H$ M(circular face), $T_1H = W(H - \bar{x})$ Eliminate W	

Question Number	Scheme	Marks
4(a)	$R \cos \alpha = mg$	M1 A1
	$R \sin \alpha = \frac{m \left(\frac{1}{4} gr \right)}{r}$ OR: $mg \sin \alpha = \frac{m \left(\frac{1}{4} gr \right)}{r} \cos \alpha$	M1A1 M2 A2
	$\tan \alpha = \frac{1}{4} *$	A1*
		(5)
4(b)	Vert equil: $S \cos \alpha - F \sin \alpha = mg$ Perp N2L: $S - mg \cos \alpha = \frac{mV^2}{r} \sin \alpha$	M1A1
	N2L towards O: $S \sin \alpha + F \cos \alpha = \frac{mV^2}{r}$ Parallel N2L: $F + mg \sin \alpha = \frac{mV^2}{r} \cos \alpha$	M1A1
	$F = \mu S$	B1
	Eliminate F , sub for trig and solve for V in terms of μ , r and g .	dM1
	$V = \sqrt{rg \frac{(1+4\mu)}{(4-\mu)}}$ oe	A1
		(7)
		(12)
	Notes	
4(a)		
	Note: For use of θ instead of α in (a) penalise only the last mark in (a). The maximum score is M1A1M1A1A0*	
M1	Resolve vertically correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation	
M1	Equation of motion horizontally correct no. of terms, condone sign errors and sin/cos confusion. V does not need to be substituted. Allow $r\omega^2$ for acceleration but not 'a'.	
A1	Correct equation. May still contain V and either form of acceleration (circular).	
OR	M2 Equation of motion down the plane correct no. of terms, condone sign errors and sin/cos confusion. A1 Correct equation with at most one error A1 Correct equation	
A1*	Correctly obtain given answer, written exactly.	
4(b)		
M1	Resolve vertically or equation of motions perpendicular. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if R from (a) is used.	

Question Number	Scheme	Marks
A1	Correct equation	
M1	Equation of motion horizontally or parallel to slope. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if R from (a) is used.	
A1	Correct equation	
B1	$F = \mu S$ seen where S is the normal reaction in (b).	
dM1	Eliminate F , sub for trig and solve for V in terms of μ , r and g . Dependent on both previous M marks.	
A1	Correct answer.	

Question number	Scheme	Marks
5(a)	$v \frac{dv}{dx} = -\frac{gR^2}{x^2}$	M1 A1
	$\int v dv = -\int \frac{gR^2}{x^2} dx$ or $\frac{1}{2}V^2 = -\int \frac{gR^2}{x^2} dx$ Or the Energy alternative below	M1
	$\frac{1}{2}v^2 = \frac{gR^2}{x} + C$	A1
	Use of $x = R, v = U$ to find C ($C = \frac{1}{2}u^2 - gR$)	M1
	$v^2 = \frac{2gR^2}{x} + U^2 - 2gR$ *	A1*
		(6)
5(b)	$\frac{1}{4}gR = \frac{2gR^2}{x} + gR - 2gR$	M1
	$x = \frac{8R}{5}$	A1
	$AB = \frac{3R}{5}$ oe	A1
		(3)
5(c)	Correct statement regarding $\frac{2gR^2}{x}$ for example <ul style="list-style-type: none"> • $\frac{2gR^2}{x} > 0$ for $x \geq R$ • $x \rightarrow \infty, \frac{2gR^2}{x} \rightarrow 0$ 	M1
	Correct reasoning. <ul style="list-style-type: none"> • $U^2 - 2gR = 0$ • $U^2 \rightarrow 2gR$ • $U^2 \geq 2gR$ 	dM1
	$U_{\text{MIN}} = \sqrt{2gR}$	A1
		(3)
		(12)
Notes		
5(a)		
M1	Equation with or without -ve sign and any derivative form for the acceleration	
A1	Correct equation with -ve sign	
M1	Separate variables and clear attempt to integrate acceleration in terms of v and x .	
A1	Correct equation; allow without C	
M1	Use of initial conditions or limits	
A1*	Given answer correctly obtained	

5(b)	
M1	Substitution of v^2 and U^2 into (a) to produce a correct equation
A1	Correct value of x
A1	cao
5(c)	
M1	Correct reasoning for the term $\frac{2gR^2}{x}$ Accept $x = \infty$, $x \rightarrow \infty$, $\frac{2gR^2}{x} = 0$
dM1	Dependent on previous M. Correct reasoning leading to correct equation or inequality.
A1	cso
ALT 5a	Energy approach must use integration
M1 A1	Energy equation with variable force. The sign may be missing for the M mark. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \int F \, dx = \int -\frac{mgR^2}{x^2} \, dx$
M1 A1	Clear attempt to integrate. Limits may be missing or incorrect. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \left[\frac{mgR^2}{x} \right]_R^x$
M1	Correct limits substituted the right way round. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \frac{mgR^2}{x} - \frac{mgR^2}{R}$
	$v^2 - U^2 = \frac{2gR^2}{x} - 2gR$
A1*	$v^2 = \frac{2gR^2}{x} + U^2 - 2gR$

Question number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \sin \theta$	M1A1A1
	$T - mg \sin \theta = \frac{mv^2}{a}$	M1A1A1
	$T = \frac{mu^2}{a} + 3mg \sin \theta^*$	A1*
		(7)
(b)	$0 = \frac{m\left(\frac{12ag}{5}\right)}{a} + 3mg \sin \theta$	M1
	$\sin \theta = -\frac{4}{5}$	A1
	$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{12ag}{5}\right) = mga \times -\frac{4}{5}$ OR $0 - mg \times -\frac{4}{5} = \frac{mv^2}{a}$	M1
	$v = 2\sqrt{\frac{ag}{5}}, 0.89\sqrt{ag}$ or better	A1
		(4)
(c)	Vertical motion $0 = \left(2\sqrt{\frac{ag}{5}} \times \frac{3}{5}\right)^2 - 2gh$ OR Energy $mgh = \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}}\right)^2 - \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2$	M1A1ft
	$h = \frac{18a}{125}$	A1
	$H, \text{ height above } O = h - a \sin \theta = \frac{18a}{125} + \frac{4a}{5}$	dM1
	$= \frac{118a}{125}, 0.94a, 0.944a$	A1
		(5)
	OR using energy from start to top	
	$mgH = \frac{1}{2}m\left\{\frac{12ag}{5} - \left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2\right\}$	M2A1ft A1
	$H = \frac{118a}{125}, 0.94a, 0.944a$	A1
		(5)
		(16)
	Notes	
6(a)		
M1	Energy equation with correct no. of terms, dim correct. May use h instead of $a \sin \theta$	
A1	Correct equation with at most one error	

A1	Correct equation
M1	Equation of motion towards O , correct no. of terms, condone sign errors and sin/cos confusion. Accept acceleration in either circular form but do not accept ' a '. The radius may be given as r .
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained and written exactly as printed.
6(b)	
M1	Put $T = 0$ and $u = 2\sqrt{\frac{3ag}{5}}$
A1	Correct value of $\sin \theta$
M1	Put $u = 2\sqrt{\frac{3ag}{5}}$ and their $\sin \theta$ into energy equation OR put $T = 0$ and their $\sin \theta$ into equation of motion
A1	Correct answer
6(c)	If an energy approach is used in (c) the equation must have 2 KE terms, one of which must have a sin/cos component included.
M1	Use vertical motion or energy to obtain an equation in h only. A component of speed must be used for either approach.
A1ft	Correct equation ft on their answer to (b).
A1	Correct value of h
dM1	Correct method to find H . Dependent on previous M.
A1	cao
	OR
M2	Complete method to obtain an equation in H only (must be using horizontal cpt of velocity at the top)
A1ft	Correct equation with at most one error
A1	Correct equation
A1	Correct answer

Question number	Scheme	Marks
7(a)	$\frac{\lambda(D-l)}{l} = mg$	M1A1
	$\frac{\lambda(2l)^2}{2l} = mg \times 3l$	M1A1A1
	$D = \frac{5l}{3}$ *	A1*
		(6)
7(b)	$mg - T = m\ddot{x}$ or $T - mg = m\ddot{x}$	M1
	$mg - \frac{3mg}{2l}(\frac{2l}{3} + x) = m\ddot{x}$ or $\frac{3mg}{2l}(\frac{2l}{3} - x) - mg = m\ddot{x}$	dM1A1
	$-\frac{3g}{2l}x = \ddot{x}$ hence SHM	A1
	period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2l}}}$ { $\omega = \sqrt{\frac{3g}{2l}}$ }	M1
	$= 2\pi\sqrt{\frac{2l}{3g}}$ *	A1*
		(6)
7(c)	$-\frac{2l}{3} = \frac{4l}{3}\cos\sqrt{\frac{3g}{2l}}t$	M1A1A1
	$t = \frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$	A1
	OR	
	Complete method $t = \frac{1}{4}2\pi\sqrt{\frac{2l}{3g}} + t_1$ where $\frac{2l}{3} = \frac{4l}{3}\sin\sqrt{\frac{3g}{2l}}t_1$	M1A1A1
	$t = \frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$ oe	A1
	OR	
	Complete method $t = \frac{1}{2}2\pi\sqrt{\frac{2l}{3g}} - t_1$ where $\frac{2l}{3} = \frac{4l}{3}\cos\sqrt{\frac{3g}{2l}}t_1$	M1A1A1
	$t = \frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$ or equivalent exact form.	A1
		(4)
		(16)
	Notes	
7(a)		
M1	Use Hooke's law in D and equate to mg	
A1	Correct equation	

M1	Energy equation with correct no. of terms. EPE of the form $\frac{\lambda x^2}{kl}, k \neq 1$
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained
7(b)	
M1	Equation of motion in a <i>general</i> position, allow a for acceleration, correct no. of terms, condone sign errors
dM1	Use Hooke's Law to sub for the tension with extension measured from the equilibrium position and allow a for acceleration
A1	Correct unsimplified equation, allow a for acceleration
A1	Correct SHM equation and conclusion. Must use \ddot{x} for acceleration and conclude SHM.
M1	Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.
A1*	Obtain the given answer for the period. Must follow from fully correct working, including N2L. At least one line of working must be seen between $\ddot{x} = -\omega^2 x$ and reaching the given answer. Eg <ul style="list-style-type: none"> • $\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\sqrt{3g}}{\sqrt{2l}}} = 2\pi \sqrt{\frac{2l}{3g}}$ • $\omega = \sqrt{\frac{3g}{2l}}$, $\text{period} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2l}{3g}}$
(c)	
M1	Complete method to find the required time. Do not ISW. For example, If the sine approach is used, it must include $\frac{1}{4}T$ + their t value for M1. If the cos approach is used with $+\frac{2l}{3}$, it must include $\frac{1}{2}T$ - their t value for M1. The correct ω must be used. For the method, condone any multiple of l for the amplitude.
A1	Equation with at most one error
A1	Correct equation
A1	Cao

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