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| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WFM01/01**

# Mathematics

## International Advanced Subsidiary/ Advanced Level
## Further Pure Mathematics F1

**You must have:**
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over*





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**1.** Given that

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}$$

where $k$ is a non-zero constant,

(a) determine the matrix **AB**

**(2)**

(b) determine the value of $k$ for which $\det(\mathbf{AB}) = 0$

**(3)**

**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that for all positive integers $n$

$$\sum_{r=1}^{n} (7r-5)^2 = \frac{n}{6}(7n+1)(An+B)$$

where $A$ and $B$ are integers to be determined.

**(6)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$f(z) = 4z^3 + pz^2 - 24z + 108$$

where $p$ is a constant.

Given that $-3$ is a root of the equation $f(z) = 0$

(a) determine the value of $p$

**(2)**

(b) using algebra, solve $f(z) = 0$ completely, giving the roots in simplest form,

**(4)**

(c) determine the modulus of the complex roots of $f(z) = 0$

**(2)**

(d) show the roots of $f(z) = 0$ on a single Argand diagram.

**(2)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 10 marks)**

4.

$$f(x) = 1 - \frac{1}{8x^4} + \frac{2}{7\sqrt{x^7}} \qquad x > 0$$

The equation $f(x) = 0$ has a single root, $\alpha$, that lies in the interval $[0.15, 0.25]$

(a) (i) Determine $f'(x)$

(ii) Explain why 0.25 cannot be used as an initial approximation for $\alpha$ in the Newton-Raphson process.

(iii) Taking 0.15 as a first approximation to $\alpha$ apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to $\alpha$
Give your answer to 3 decimal places.

**(5)**

(b) Use linear interpolation once on the interval $[0.15, 0.25]$ to find another approximation to $\alpha$
Give your answer to 3 decimal places.

**(3)**

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**Question 4 continued**

**Question 4 continued**


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Question 4 continued

**(Total for Question 4 is 8 marks)**

**5.** The quadratic equation

$$4x^2 + 3x + k = 0$$

where $k$ is an integer, has roots $\alpha$ and $\beta$

(a) Write down, in terms of $k$ where appropriate, the value of $\alpha + \beta$ and the value of $\alpha\beta$

**(2)**

(b) Determine, in simplest form in terms of $k$, the value of $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2}$

**(4)**

(c) Determine a quadratic equation which has roots

$$\frac{\alpha}{\beta^2} \text{ and } \frac{\beta}{\alpha^2}$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integer values in terms of $k$

**(3)**

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Question 5 continued

**Question 5 continued**

Question 5 continued

**(Total for Question 5 is 9 marks)**

**6.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The rectangular hyperbola $H$ has equation $xy = 20$

The point $P\left(2t\sqrt{a}, \frac{2\sqrt{a}}{t}\right)$, $t \neq 0$, where $a$ is a constant, is a general point on $H$

(a) State the value of $a$

**(1)**

(b) Show that the normal to $H$ at the point $P$ has equation

$$ty - t^3x - 2\sqrt{5}\left(1 - t^4\right) = 0$$

**(4)**

The points $A$ and $B$ lie on $H$

The point $A$ has parameter $t = c$ and the point $B$ has parameter $t = -\frac{1}{2c}$, where $c$ is a constant.

The normal to $H$ at $A$ meets $H$ again at $B$

(c) Determine the possible values of $c$

**(4)**

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Question 6 continued

**Question 6 continued**

Question 6 continued

(Total for Question 6 is 9 marks)

**7.** (i)

$$\mathbf{P} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents a geometrical transformation $U$

(a) Describe $U$ fully as a single geometrical transformation.

**(2)**

The transformation $V$, represented by the $2 \times 2$ matrix **Q**, is a rotation through 240° anticlockwise about the origin followed by an enlargement about (0, 0) with scale factor 6

(b) Determine the matrix **Q**, giving each entry in exact numerical form.

**(2)**

Given that $U$ followed by $V$ is the transformation $T$, which is represented by the matrix **R**

(c) determine the matrix **R**

**(2)**

(ii) The transformation $W$ is represented by the matrix

$$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}$$

Show that there is a real number $\lambda$ for which $W$ maps the point $(\lambda, 1)$ onto the point $(4\lambda, 4)$, giving the exact value of $\lambda$

**(5)**

Question 7 continued

Question 7 continued

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**Question 7 continued**

**(Total for Question 7 is 11 marks)**

**8.** A parabola $C$ has equation $y^2 = 4ax$ where $a$ is a positive constant.

The point $S$ is the focus of $C$

The line $l_1$ with equation $y = k$ where $k$ is a positive constant, intersects $C$ at the point $P$

(a) Show that

$$PS = \frac{k^2 + 4a^2}{4a}$$

**(3)**

The line $l_2$ passes through $P$ and intersects the directrix of $C$ on the $x$-axis.

The line $l_2$ intersects the $y$-axis at the point $A$

(b) Show that the $y$ coordinate of $A$ is $\dfrac{4a^2k}{k^2 + 4a^2}$

**(3)**

The line $l_1$ intersects the directrix of $C$ at the point $B$

Given that the areas of triangles $BPA$ and $OSP$, where $O$ is the origin, satisfy the ratio

$$\text{area } BPA : \text{area } OSP = 4k^2 : 1$$

(c) determine the exact value of $a$

**(5)**

**Question 8 continued**

**Question 8 continued**





P 7 2 4 6 4 A 0 2 8 3 2

**Question 8 continued**

**(Total for Question 8 is 11 marks)**

9. Prove by induction that for all positive integers $n$

$$\sum_{r=1}^{n} \log(2r-1) = \log\left(\frac{(2n)!}{2^n n!}\right)$$

**(6)**

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**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 6 marks)**

**TOTAL FOR PAPER IS 75 MARKS**