



# Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level  
In Pure Mathematics P1 (WMA11) Paper 01

## **Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at [www.edexcel.com](http://www.edexcel.com) or [www.btec.co.uk](http://www.btec.co.uk). Alternatively, you can get in touch with us using the details on our contact us page at [www.edexcel.com/contactus](http://www.edexcel.com/contactus).

## **Pearson: helping people progress, everywhere**

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: [www.pearson.com/uk](http://www.pearson.com/uk)

Summer 2022

Question Paper Log Number P69458A

Publications Code WMA11\_01\_2206\_MS

All the material in this publication is copyright

© Pearson Education Ltd 2022

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

### Method mark for solving 3 term quadratic:

#### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### 2. Formula

Attempt to use correct formula (with values for  $a$ ,  $b$  and  $c$ ).

#### 3. Completing the square

Solving  $x^2 + bx + c = 0$ :  $(x \pm \frac{b}{2})^2 \pm q \pm c$ ,  $q \neq 0$ , leading to  $x = \dots$

### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

### Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

### Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question	Scheme	Marks
1	$\int 10x^5 + 6x^3 - \frac{3}{x^2} dx = 10 \times \frac{x^6}{6} + 6 \times \frac{x^4}{4} - 3 \times \frac{x^{-1}}{-1} (+c).$	M1 A1
	$= \frac{5x^6}{3} + \frac{3x^4}{2} + \frac{3}{x} + c$	A1A1
		(4)
<b>(4 marks)</b>		
<b>Notes:</b>		

**M1:** For raising any power by 1, e.g.  $x^5 \rightarrow ..x^6$ ,  $x^3 \rightarrow ..x^4$  or  $x^{-2} \rightarrow ..x^{-1}$ .

Accept unprocessed indices such as  $x^{5+1}$  etc

**A1:** For two of the three terms in  $x$  correctly integrated (but may be left unsimplified). See below

Accept for this mark terms like  $10 \times \frac{x^6}{6}$  and  $-3 \times \frac{x^{-1}}{-1}$

This may be implied by a correct simplified answer.

Do NOT accept for this mark "unprocessed terms" such as  $10 \times \frac{x^{5+1}}{5+1}$

**A1:** For two correct and simplified terms in  $x$  all on one line.

Accept for this mark equivalent terms such as  $3x^{-1}$  for  $\frac{3}{x}$  and  $1.5x^4$  for  $\frac{3x^4}{2}$

Do NOT accept for this mark terms like  $-3 \times \frac{x^{-1}}{-1}$  and  $1.67x^6$

**A1:** Fully correct and simplified with  $+c$  all on one line.

Accept simplified equivalents, e.g.  $3x^{-1}$  for  $\frac{3}{x}$ .

Do NOT accept with spurious symbols like  $\left(\frac{5x^6}{3} + \frac{3x^4}{2} + \frac{3}{x} + c\right) dx$  or  $\int \left(\frac{5x^6}{3} + \frac{3x^4}{2} + \frac{3}{x} + c\right)$

Do NOT allow if they then go on and multiply to get rid of fractions

Question	Scheme	Marks
2(a)	$AB = 21 \text{ cm}, BC = 13 \text{ cm}, \angle BAC = 25^\circ, \angle ACB = x^\circ$	
	$\frac{\sin x^\circ}{21} = \frac{\sin 25^\circ}{13}$ o.e	M1
	$\sin x^\circ = 0.6827$ (awrt)	A1
		(2)
(b)	$\sin^{-1}(0.6827) = \dots(43.05^\circ)$	M1
	$(AC < AB \text{ so } \angle ABC < \angle ACB \text{ so})$ required angle is $180^\circ - \sin^{-1}(0.6827) = \dots$	M1
	So $x =$ awrt 136.95	A1
		(3)
		(5 marks)
<b>Notes:</b>		
<b>Condone the omission of the <math>^\circ</math> symbol. Mark (a) and (b) as one</b>		

(a)

**M1:** A correct statement of the sine rule with sides and angles in the correct position.

Implied by  $\sin x^\circ =$  awrt 0.68

**A1:**  $\sin x^\circ =$  awrt 0.6827

ISW for instance if they go on to find the value of  $x$  in (a)

(b)

**M1:** Applies inverse sine to their value found in part (a) to find the angle in degrees (using degree mode) correct to nearest degree. For a correct (a) awrt  $43^\circ$  is sufficient

Implied by a correct answer for their value of  $\sin x$ , either for the acute or the obtuse angle..

**M1:** Attempts to find the correct angle,  $180^\circ - \arcsin "0.6827"^\circ$ . Award even if other angles are given

No reasoning need be given, but may see diagram drawn, size of angles, or comparison of sides used.

**A1:** awrt 136.95 but it is A0 if two angles are given

.....  
There are alternative methods so look at the candidates work carefully.

E.g.

Uses the cosine rule once to find  $AC$

$$13^2 = 21^2 + y^2 - 2 \times 21 \times y \cos 25$$

$$y^2 - 38.1..y + 272 = 0$$

$$y = 9.533\dots$$

...and then a second time to find  $x$

$$\cos x = \frac{13^2 + 9.533^2 - 21^2}{2 \times 13 \times 9.533} \Rightarrow x = 136.95^\circ$$

In this solution both M's in part (b) are scored together.

To score marks in (a) (even though they haven't used the sine rule) they would need to simply find  $\sin$  (their 136.95) for M1, and A1 for an accurate answer.

Question	Scheme	Marks
3(i)	Writes $\sqrt{180}$ as $6\sqrt{5}$ or $\sqrt{80}$ as $4\sqrt{5}$	M1
	Concludes working $\frac{6\sqrt{5}-4\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} = 2$	A1
		(2)
(ii)	$\frac{4\sqrt{5}-5}{7-3\sqrt{5}} = \frac{4\sqrt{5}-5}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}} = \dots$	M1
	$= \frac{28\sqrt{5}-35+12(\sqrt{5})^2-15\sqrt{5}}{49-9 \times 5}$	dM1
	$= \frac{25}{4} + \frac{13}{4}\sqrt{5}$	A1
		(3)
		(5 marks)
<b>Notes: This is a non calculator question and all stages of working must be shown</b>		

**(i) Main method**

**M1:** Writes  $\sqrt{180}$  as  $6\sqrt{5}$  or  $\sqrt{80}$  as  $4\sqrt{5}$ .

Additional lines of working may be seen but they are not required for this mark

**A1:** Correct work leading to the answer 2. The M mark must have been awarded.

The solution shown in the main mark scheme is the minimum evidence required.

A valid alternative is  $\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} = \frac{6\sqrt{5}-4\sqrt{5}}{\sqrt{5}} = \left(\frac{6\sqrt{5}}{\sqrt{5}} - \frac{4\sqrt{5}}{\sqrt{5}}\right) = 6-4=2$

**(i) Alt method I**  $\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} = \frac{\sqrt{180}}{\sqrt{5}} - \frac{\sqrt{80}}{\sqrt{5}} = \sqrt{36} - \sqrt{16} = 6-4=2$

**M1:** Writes  $\frac{\sqrt{180}}{\sqrt{5}}$  as  $\sqrt{36}$  or  $\frac{\sqrt{80}}{\sqrt{5}}$  as  $\sqrt{16}$

Additional lines of working may be seen but they are not required

**A1:** Correct work leading to the answer 2. The solution shown below is the minimum evidence required

E.g.  $\frac{\sqrt{180}}{\sqrt{5}} - \frac{\sqrt{80}}{\sqrt{5}} = \sqrt{36} - \sqrt{16} = 6-4=2$

**(i) Alt method II**  $\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{900}-\sqrt{400}}{5} = \frac{30-20}{5} = 2$

**M1:** Multiplies numerator and denominator by  $\sqrt{5}$  and writes  $\sqrt{180} \times \sqrt{5}$  as  $\sqrt{900}$  or  $\sqrt{80} \times \sqrt{5}$  as  $\sqrt{400}$

**A1:** Correct work leading to the answer 2

**Do NOT allow solutions that rely on huge jumps. The M marks must be awarded as above.**

For example

$$\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} = \frac{(\sqrt{180}-\sqrt{80})\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{30-20}{5} = 2 \text{ scores M0 A0}$$

without sight of  $\sqrt{180}\times\sqrt{5}$  as  $\sqrt{900}$  or  $\sqrt{80}\times\sqrt{5}$  as  $\sqrt{400}$  o.e.

Or

$$\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} = 2 \text{ scores M0 A0 without sight of } \sqrt{180} \text{ as } 6\sqrt{5} \text{ or } \sqrt{80} \text{ as } 4\sqrt{5}$$

(ii)

**Main method**

**M1:** Correct attempt to rationalise. Look for attempt at multiplying numerator and denominator by  $7+3\sqrt{5}$   
The multiplications need not be carried out for this mark.

**dM1:** It is dependent upon the previous mark. Scored for

- Correct expression for denominator (without surds) Accept as shown, or as  $7^2 - 3^2 \times 5$  or just 4
- and attempts to multiply out the numerator with the sight of 4 terms . Condone slips here. See \*\*\*

**A1:** Correct answer any way around. Allow  $6.25+3.25\sqrt{5}$

Do NOT allow in the form  $\frac{25+13\sqrt{5}}{4}$  but accept if followed by "hence  $a = \frac{25}{4}, b = \frac{13}{4}$ "

**(ii) Alt method**

**M1:** Sets  $\frac{4\sqrt{5}-5}{7-3\sqrt{5}} = a+b\sqrt{5} \Rightarrow 4\sqrt{5}-5 = (a+b\sqrt{5})(7-3\sqrt{5})$  and attempts to set up simultaneous

equations in  $a$  and  $b$ . FYI the correct equations are  $7a-15b = -5, 7b-3a = 4$

**dM1:** Attempts to solve their equations. They must have at least one correct equation

**A1:** Correct answer. Must be  $\frac{25}{4} + \frac{13}{4}\sqrt{5}$  and not **just** values of  $a$  and  $b$

The following solution is a minimally acceptable solution for 3 marks, the first M1 being implied.

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}} = \frac{(4\sqrt{5}-5)\times(7+3\sqrt{5})}{4} = \frac{28\sqrt{5}-35+60-15\sqrt{5}}{4} = \frac{13}{4}\sqrt{5} + \frac{25}{4} \quad ***$$

Question	Scheme	Marks	
4(i)		Correct shape, translated down.	<b>B1</b>
		Correct horizontal asymptote labelled	<b>B1</b>
		Correct <b>maximum</b> and <b>minimum</b> points labelled	<b>B1</b>
			<b>(3)</b>
(ii)		Correct shape, reflected in y axis	<b>B1</b>
		Correct horizontal asymptote labelled	<b>B1</b>
		Correct <b>maximum</b> and <b>minimum</b> points labelled.	<b>B1</b>
			<b>(3)</b>
<b>(6 marks)</b>			
<b>Notes:</b>			

In both parts be tolerant with “pen slips” for the asymptotes. Judge the intent of the shape.

**(i) There is no MR for sketching  $y = f(x - 2)$ . These marks are independent of each other**

**B1:** Same shape as the original graph but translated vertically downwards.

Do not consider any coordinates here.

Look for a minimum to the left of the  $y$  axis and a maximum to the right

Be tolerant of the asymptote not being **exactly** at the same level at either end of the curve but withhold the mark if it is intentionally different. Condone slight upturns at either end

**B1:** For  $y = -1$  labelled as the horizontal asymptote. It must be below the  $x$  axis and the intention must be for ends to be at the same height/level. The intention must be for the curve to be asymptotic here (at both ends)

**B1:** For the coordinates of the maximum and minimum points in the correct positions. E.g.  $\left(\frac{3}{2}, 0\right)$  must be

on the positive  $x$ -axis and  $(-1, -2)$  is in quadrant 3. Note that  $P$  and  $Q$  don't need to be seen, just their coordinates. Coordinates may be given in the body of the script but must match with maximum and minimum points marked in some way on the graph

**(ii) There is no MR for sketching  $y = -f(x)$  These marks are independent of each other**

**B1:** For the correct shape, a reflection in the  $y$ -axis. The intention should be for the asymptotes to be at the same level but be tolerant of slips. Look for the image  $Q'$  a maximum being to the left of  $y$ -axis in quadrant two and  $P'$  a minimum on the positive  $x$ - axis.

**B1:** For  $y = 1$  labelled as the horizontal asymptote. It must be above the  $x$  axis and the intention must be for ends to be at the same height/level. The curve must be asymptotic here (at both ends)

**B1:** For the coordinates of the maximum and minimum points in correct positions. E.g.  $P'$  must be on the  $x$ -axis although  $P$  and  $Q$  don't need to be seen, just their coordinates. Coordinates may be given in the body of the script but must match with maximum and minimum points marked in some way on the graph.

Question	Scheme	Marks
<b>5(a)</b>	E.g. $12 - a(x+2)^2$ or $a(x-1)(x+5)$ or $y = ax^2 + bx + c \Rightarrow 4a - 2b + c = 12$ and $25a - 5b + c = 0$	<b>M1A1</b>
	E.g. $0 = 12 - a(-5+2)^2 \Rightarrow a = \dots$ or $12 = a(-2-1)(-2+5) \Rightarrow a = \dots$ or $2a(-2) + b = 0, 25a - 5b + c = 0, 4a - 2b + c = 12 \Rightarrow a = \dots, b = \dots, c = \dots$	<b>dM1</b>
	$12 - \frac{4}{3}(x+2)^2$ or $-\frac{4}{3}(x-1)(x+5)$ or $-\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ oe	<b>A1</b>
		<b>(4)</b>
<b>(b)</b>	Gradient of $l_2$ is $\frac{-5}{4}$ o.e.	<b>M1</b>
	Equation of $l_2$ is $y = -\frac{5}{4}(x+5)$	<b>M1</b>
	$y = -\frac{5}{4}x - \frac{25}{4}$	<b>A1</b>
		<b>(3)</b>
<b>(c)</b>	For two of $y \geq -\frac{5}{4}x - \frac{25}{4}$ ; $y \geq \frac{4}{5}x$ ; $y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ Or with strict inequalities	<b>M1</b>
	For all three of $y \geq -\frac{5}{4}x - \frac{25}{4}$ ; $y \geq \frac{4}{5}x$ ; $y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ Or with strict inequalities	<b>A1ft</b>
		<b>(2)</b>
		<b>(9 marks)</b>
<b>Notes:</b>		

**(a) Just the expression for  $f(x)$  is required but you may see  $y = \dots$  or  $f(x) = \dots$  which is fine**

**M1:** For knowing a quadratic form and using two correct pieces of information. This can be scored by

- using the maximum value occurs at  $(-2, 12)$  to state the form  $12 \pm a(x \pm 2)^2$
- use symmetry, usually with roots of  $-5$  and  $1$  to state the form  $a(x \pm 1)(x \pm 5)$
- using the general form  $y = ax^2 + bx + c$  and substituting in two points, usually  $(-2, 12)$  and  $(-5, 0)$ . An alternative is using just one point and setting  $\frac{dy}{dx} = 0$  at  $x = -2 \Rightarrow -4a + b = 0$

**Condone for each of these: (1) slips (e.g incorrect squaring) or (2) the use of  $a = \pm 1$**

**A1:** For a **correct equation/expression set up** with the correct unknown constants included. It is for getting in a position where only one more equation or constant, usually the value of  $a$ , needs to be found. If  $a$  is set =  $\pm 1$  then this mark cannot be awarded

Usually scored for one of

- using the maximum of  $(-2,12)$  to state the form  $(y) = 12 \pm a(x+2)^2$
- using the symmetry to deduce roots, usually  $-5$  and  $1$  to state the form  $(y) = a(x-1)(x+5)$
- using the general form  $(y) = ax^2 + bx + c$  and substituting in two points, usually  $(-2,12)$  and  $(-5,0)$  to form two different equations in  $a, b$  and  $c$ . Award for example for  $4a - 2b + c = 12$  and  $25a - 5b + c = 0$

**dM1:** For a full method of finding  $f(x)$  in any allowable form.

- using the form  $y = 12 \pm a(x+2)^2$  with usually  $(-5,0)$  to find  $a$
- using the form  $y = a(x-1)(x+5)$  with usually  $(-2,12)$  to find  $a$
- using the general form  $y = ax^2 + bx + c$  scored for forming three equations and solving to find values for  $a, b$  and  $c$ . E.g Uses  $(-2,12)$  and  $(-5,0)$  in  $y = f(x)$  and then uses  $2ax + b = 0$  at  $x = -2$

**A1:** Correct equation or expression for  $f(x)$  or  $y$ . Accept in any equivalent form

- $(f(x) =) 12 - \frac{4}{3}(x+2)^2$
- $(y =) -\frac{4}{3}(x-1)(x+5)$
- $(y =) -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$

(b)

**M1:** Applies perpendicular condition to find gradient of  $l_2$

Look for  $\frac{-5}{4}$ ,  $-1.25$  or it may be implied by an equation for  $l_2$  of  $y = \frac{-5}{4}x + \dots$

**M1:** Uses their changed gradient with  $(-5,0)$  to find the equation of the line. Look for  $y = "m_n"(x+5)$

If they use the form  $y = mx + c$  with  $(-5,0)$  they must proceed as far as  $c = \dots$

**A1:** Correct equation  $y = -\frac{5}{4}x - \frac{25}{4}$  o.e. Allow exact equivalents but must be in the form  $y = mx + c$

(c)

**M1:** For identifying any two of the three restrictions given in the scheme. FT on their answers to (a) and (b) where (a) is a quadratic and (b) is linear. Allow for this mark only the inequality for the quadratic to be  $y \leq ax^2 + bx + c$  or  $y \leq f(x)$  if that is what they identify as the curve. They may combine two of these

as one statement, e.g.  $\frac{4}{5}x \leq y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ , which is fine (and scores the M1). Use of  $R$

instead of  $y$  is M0 A0

**A1ft:** For all three of the restrictions given in the scheme, follow through their answers to (a) and (b) Accept with strict inequalities, but should be consistent in all inequalities.

The use of set notation is fine  $\left\{ (x, y) : \frac{4}{5}x \leq y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3} \right\} \cap \left\{ (x, y) : y \geq -\frac{5}{4}x - \frac{25}{4} \right\}$

Spurious additional restrictions such as  $x \geq -5$  may be seen and should not be marked as incorrect unless the additional restriction gives further restrictions on the region  $R$ , e.g.  $y \geq 0$

Do not ISW here. Mark their final answer.

Question	Scheme	Marks
<b>6(a)</b>	$2xy - 3x^2 = 50; y - x^3 + 6x = 0$	
	$\Rightarrow 2x(x^3 - 6x) - 3x^2 = 50$	<b>M1</b>
	$\Rightarrow 2x^4 - 12x^2 - 3x^2 - 50 = 0 \Rightarrow 2x^4 - 15x^2 - 50 = 0^*$ CSO	<b>A1*</b>
		<b>(2)</b>
<b>(b)</b>	$\Rightarrow (2x^2 + 5)(x^2 - 10) = 0 \Rightarrow x^2 = \dots$	<b>M1</b>
	So $x^2 = 10$	<b>A1</b>
	$\Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = \dots$	<b>M1</b>
	one solution pair is $x = \sqrt{10}, y = 4\sqrt{10}$	<b>A1</b>
	Solutions are $x = \sqrt{10}, y = 4\sqrt{10}$ and $x = -\sqrt{10}, y = -4\sqrt{10}$ CSO	<b>A1</b>
		<b>(5)</b>
		<b>(7 marks)</b>

**(a)**

**M1:** Attempts to substitute their expression for  $y$  from rearranging the second equation into the first equation. More difficult approaches are possible, but all should lead to an equation in just  $x$

E.g. 1 substituting their expression for  $y$  from rearranging the first equation into the second equation

E.g. 2 multiplying the second equation by  $2x$  and subtracting

**A1\*:** Correct work leading to the given answer. Expect to see correct bracketing etc and at least one intermediate line with the two  $x^2$  terms initially uncollected. See scheme

**(b) This is a non calculator question so look for evidence of calculations**

**M1:** Uses a non calculator method to solve the given quadratic in  $x^2$ .

Allow factorisation, the quadratic formula or completing the square.

You may see a substitution, say  $u = x^2$ . In this case it is for solving to find  $u$

Condone substitutions  $y = x^2$  or  $x = x^2$  if it is clear what is meant.

E.g.  $2x^4 - 15x^2 - 50 = 0 \Rightarrow x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4 \times 2 \times -50}}{4} = 10, -\frac{5}{2}$  scores 0 unless recovered to  $x^2 = 10$  or  $x = \sqrt{10}$

**A1:** A correct solution for  $x$  or  $x^2$  e.g.  $x^2 = 10$ ,  $u = 10$   $x = \pm\sqrt{10}$ ,  $x = \sqrt{10}$  etc but not  $x = 10$

Candidates cannot just write down  $2x^4 - 15x^2 - 50 = 0 \Rightarrow x = \pm\sqrt{10}$

**M1:** Uses at least one  $x$  value of the form  $p\sqrt{a}$ ,  $a > 0$  and uses a non calculator method to find a value for  $y = q\sqrt{a}$ . Must use  $x$  correctly, not  $x^2$  in place of  $x$ . It is not a dependent mark so if  $x = \sqrt{10}$  followed by a correct non calculator method you may award the mark. Some calculations must be seen, not just a written down answer. So  $x = \sqrt{10} \Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = 4\sqrt{10}$  is acceptable

**A1:** At least one correct pair of solutions. Need not be fully simplified but should be single terms, so accept e.g.  $\left(\sqrt{10}, \frac{40}{\sqrt{10}}\right)$  if they use the other equation to find  $y$ .

So a mark of 00110 is possible for candidates who show evidence for  $y$  but not for  $x$ .

**A1:** CSO. All previous marks must have been awarded in part (b). Both pairs of solutions correct and simplified with no incorrect extra answers given. Must be clearly paired so do not accept  $x = \pm\sqrt{10}$ ,  $y = \pm 4\sqrt{10}$  unless clear pairings have been given elsewhere.

Question	Scheme	Marks
<b>7(a)</b>	$f'(x) = 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \Rightarrow f''(x) = \dots x^{-\frac{1}{2}-1} + \dots x^{-2-1}$	<b>M1</b>
	$\Rightarrow f''(x) = 2 \times -\frac{1}{2} x^{-\frac{3}{2}} + -2Ax^{-3} = -x^{-\frac{3}{2}} - 2Ax^{-3}$	<b>A1</b>
	$f''(4) = 0 \Rightarrow -4^{-\frac{3}{2}} - 2A \times 4^{-3} = 0 \Rightarrow A = \dots$	<b>dM1</b>
	$-\frac{1}{8} - \frac{2A}{64} = 0 \Rightarrow A = -4$	<b>A1</b>
		<b>(4)</b>
<b>(b)</b>	$f(x) = \int 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \, dx = \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{Ax^{-2+1}}{-2+1} + 3x(+c)$	<b>M1</b>
	$= 4x^{\frac{1}{2}} - \frac{A}{x} + 3x(+c)$	<b>A1ft</b>
	$f(12) = 8\sqrt{3} \Rightarrow 4\sqrt{12} - \frac{A}{12} + 36 + c = 8\sqrt{3} \Rightarrow c = \dots$	<b>dM1</b>
	$c = 8\sqrt{3} - 4\sqrt{12} - 36 - \frac{4}{12} = -\frac{109}{3}$ or follow through $c = \frac{A}{12} - 36$	<b>A1ft</b>
	So $f(x) = 4x^{\frac{1}{2}} + \frac{4}{x} + 3x - \frac{109}{3}$ oe	<b>A1</b>
	<b>(5)</b>	
		<b>(9 marks)</b>
<b>Notes: Mark parts (a) and (b) together</b>		

(a)

**M1:** Correct method of differentiation, at least one **correct** power reduced by 1.

The indices or coefficients do not have to be processed/simplified.

**A1:** Correct differentiation, need not be simplified, but the indices and coefficients must be processed correctly.

**dM1:** Sets  $f''(4) = 0$  and proceeds to find a value for  $A$ . It is dependent upon the previous M1

Don't be overly concerned with the mechanics of the solution here

**A1:**  $A = -4$

(b)

**M1:** Attempts to integrate  $f'(x)$ . Look for a **correct** power increased by 1 on at least one term. The indices and coefficients do not have to be processed/simplified. Constant of integration is not needed for this mark.

**A1ft:** Correct integration, either with  $A$  or follow through their value of  $A$ . The indices and coefficients must be processed correctly. Constant of integration is not needed for this mark.

**dM1:** Sets  $f(12) = 8\sqrt{3}$  and proceeds to find a value for a constant of integration " $c$ " using their value of  $A$ . It is dependent upon the previous M1

**A1ft:** Correct value for  $c$  found. Follow through their value of  $A$ , so  $c = \frac{A}{12} - 36$ .

Accept awrt 1 dp

**A1:** Correct answer,  $4x^{\frac{1}{2}} + \frac{4}{x} + 3x - \frac{109}{3}$ , not follow through.

If they go on to multiply by 3 etc it is A0

Question	Scheme	Marks
<b>8(a)</b>	With $\theta$ being the angle subtended by arc $AB$ and $\phi$ being the angle subtended by arc $CD$	
	$15 = 9 \times \theta \Rightarrow \theta = \frac{5}{3} = (1.67)$	<b>M1</b>
	Therefore $\phi = \frac{2\pi}{3} - \frac{5}{3} = (0.4277\dots)$	<b>dM1</b>
	So length of arc $CD = 84 \times \left(\frac{2\pi}{3} - \frac{5}{3}\right) = 35.929\dots = 35.9 \text{ cm (1 d.p.)}^* \text{ CSO}$	<b>A1*</b>
		<b>(3)</b>
<b>(b)</b>	Perimeter = $3 \times (15 + 35.9\dots) + 6 \times (84 - 9)$	<b>M1</b>
	= awrt 603 cm (602.787\dots)	<b>A1</b>
		<b>(2)</b>
<b>(c)</b>	FOR EXAMPLE Area of a "blade" is $\frac{1}{2} \times 84^2 \times \left(\frac{2\pi - 5}{3}\right) = \text{awrt (1510)}$	<b>M1</b>
	Area of sector of inner circle between "blades" is $\frac{1}{2} \times 9^2 \times \frac{5}{3} = (67.5)$	<b>dM1</b> <b>A1</b>
	Total area is $3 \left( \frac{1}{2} \times 84^2 \times \left(\frac{2\pi}{3} - \frac{5}{3}\right) + \frac{1}{2} \times 9^2 \times \frac{5}{3} \right) = \dots(4729.577764 \text{ cm}^2)$	<b>ddM1</b>
	So area is awrt 0.473 m <sup>2</sup> or awrt 4730 cm <sup>2</sup>	<b>A1</b>
		<b>(5)</b>
		<b>(10 marks)</b>
<b>Notes:</b>		

**(a)**

**M1:** Correct use of the arc length formula to find the angle subtended by arc  $AB$ .

Attempts  $15 = 9 \times \theta \Rightarrow \theta = \dots$  Don't be concerned by what the angle is called

**dM1:** Correct method to find the angle subtended by arc  $CD$  using their angle for arc  $AB$ .

Note that  $\phi = \frac{1}{3} \left( 2\pi - 3 \times \frac{5}{3} \right)$  is also correct. It is dependent upon the previous M

**A1\*:** CSO Arrives at 35.9 with a correct value to at least 2 d.p. (rounded or truncated) seen first.

Alternatively sight of  $84 \times \left(\frac{2\pi}{3} - \frac{5}{3}\right)$  or  $84 \times \text{awrt } 0.4277$  followed by  $35.9(\text{cm})$  is fine

Note that there are equivalent methods such as  $84 \times \frac{2\pi}{3} - 84 \times \frac{5}{3} = 35.9$  or  $\frac{2\pi}{3} \times 84 - 140 = 35.9$

**(b)**

**M1:** Correct method to find the perimeter, it should include all six arcs and radial edges.

Look for  $3 \times 15 + 3 \times 35.9 + 6 \times \dots$  If no method is seen it is implied by awrt 603

A1: For awrt 603 (cm). The units need not be given.

(c) This part is now being marked M1 dM1 A1 ddM1 A1

Please look through all of the solution first. The marks can be awarded in the following way.

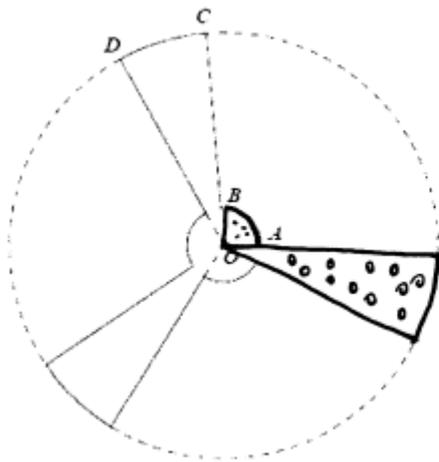
M1: A correct attempt at any relevant area

dM1: A correct attempt at a corresponding area that can be combined with the first area in some way to find the area of the fan. FT on angles found in part (a). Dependent upon previous mark

A1: Both areas correct. They do not need to be calculated but the angles must be correct to 3sf

ddM1: A correct combination of areas to find the area of the fan

A1: awrt 0.473 m<sup>2</sup> or awrt 4730 cm<sup>2</sup>. Must include the units. ISW after a correct answer



Main method

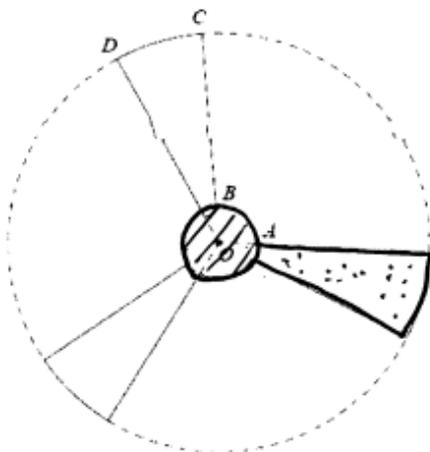
∴ M1: One relevant area eg  $\frac{1}{2} \times 9^2 \times \frac{5}{3}$

∴ dM1: Corresponding area  $\frac{1}{2} \times 84^2 \times \left(\frac{2\pi}{3} - \frac{5}{3}\right)$

A1: Both correct

ddM1: 3x  + 3x 

A1: Awrt 0.473 m<sup>2</sup> or 4730 cm<sup>2</sup>



Alt I

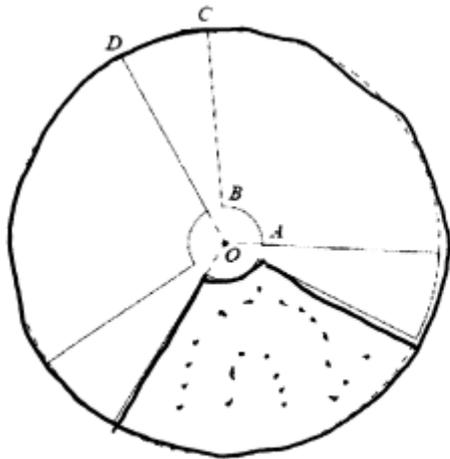
⊗ M1: One relevant area eg  $\pi \times 9^2$

∴ dM1: Corresponding area  $\frac{1}{2} \times 84^2 \left(\frac{2\pi}{3} - \frac{5}{3}\right) - \frac{1}{2} \times 9^2 \times \left(\frac{\pi}{3} - \frac{5}{3}\right)$

A1: Both correct

ddM1: ⊗ + 3x 

A1: Awrt 0.473 m<sup>2</sup> or 4730 cm<sup>2</sup>



Alt II



M1: one relevant area Eg  $\pi \times 8^2$

dM1: Corresponding area  $\frac{1}{2} \times 8^2 \times \frac{5}{3} - \frac{1}{2} \times 9^2 \times \frac{5}{3}$

A1: Both correct

ddM1:  $\circ - 3 \times$

A1: Award 0.473 m<sup>2</sup> or 4730 cm<sup>2</sup>

Variations are possible, e.g.  $3 \times$  area of blades (inc. circle) + area circle - area of blades within the circle, but these can be marked according to the scheme.

Question	Scheme	Marks
9(a)	(i) $2p$	B1
	(ii) $-p$	B1
	(iii) $3-p$	B1
		(3)
(b)		Correct shape, same height starting at O, scaling may be incorrect.
		Two repeats of the sinx graph each side
		(2)
(c)	For $x = \frac{\alpha}{2}$	B1
	Attempt at second root E.g. $x = \frac{180^\circ - \alpha}{2}$	M1
	$x = 90^\circ - \frac{\alpha}{2}$	A1
		(3)
		(8 marks)
Notes:		

(a)

(i) B1: For  $2p$ . Condone  $p + p$ . Award when  $2p$  is the y coordinate of a coordinate pair. E.g.  $(180 - \alpha, 2p)$

(ii) B1: For  $-p$ . Award when  $-p$  is the y coordinate of a coordinate pair. E.g.  $(\alpha - 180, -p)$

(iii) **B1:** For  $3 - p$ . Award when  $3 - p$  is the  $y$  coordinate of a coordinate pair. E.g  $(180 + \alpha, 3 - p)$

(b)

**M1:** Same shape, starting at  $O$ , and same height as original graph, but the scaling may be incorrect.

Mark the intention. For example their graph may be slightly taller or shorter than the given curve.

One complete sine curve on just one side of the  $y$ -axis is sufficient for the method mark.

Also allow this mark when candidates use Diagram 1 and mark the  $x$  - intercepts as below.

If they ignore the given Diagram and use their own axes you must be convinced their sketch for  $y = \sin 2x$  is the same eight / amplitude as  $y = \sin x$  but a different scaling / frequency. So you must be able to see some numbers marked on the axes or graph

**A1:** Correct sketch. Look for intersections at  $(\pm 360^\circ, 0)$  and  $(\pm 180^\circ, 0)$  in addition to the above

(c)

**B1:**  $x = \frac{\alpha}{2}$  given as one solution.

**M1:** A second solution which may not be in the given range. Any of  $\frac{180^\circ - \alpha}{2}$ ,  $90^\circ - \frac{\alpha}{2}$ ,  $180^\circ + \frac{\alpha}{2}$ ,

$-90^\circ - \frac{\alpha}{2}$ ,  $-180^\circ + \frac{\alpha}{2}$ ,  $360^\circ + \frac{\alpha}{2}$  are examples of using the symmetry to find a second solution.

**A1:**  $x = 90^\circ - \frac{\alpha}{2}$  or  $\frac{180^\circ - \alpha}{2}$  as second solution with no additional solutions within the given range.

Question	Scheme	Marks
<b>10(a)</b>	$\frac{dy}{dx} = \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2}$	<b>M1</b> <b>A1</b>
		<b>(2)</b>
<b>(b)</b>	At $x = -\frac{7}{2}$ , $\frac{dy}{dx} = \frac{6}{7}\left(-\frac{7}{2}\right)^2 + \frac{2}{7}\left(-\frac{7}{2}\right) - \frac{5}{2} = \dots (=7)$	<b>M1</b>
	So at $B$ we know $\frac{dy}{dx} = -\frac{1}{7}$	<b>M1</b>
	hence $\frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} = -\frac{1}{7}$	<b>dM1</b>
	$\Rightarrow 12x^2 + 4x - 35 = -2 \Rightarrow 12x^2 + 4x - 33 = 0^*$	<b>A1*</b>
		<b>(4)</b>
<b>(c)</b>	E.g. $12x^2 + 4x - 33 = 0 \Rightarrow (2x-3)(6x+11) = 0 \Rightarrow x = \dots$	<b>M1</b>
	From graph we can see the $x$ coordinate is positive, so $x = \frac{3}{2}$ at $B$	<b>A1</b>
		<b>(2)</b>
<b>(d)</b>	Equation of $l$ is $y = -\frac{1}{7}x - 1$	<b>M1</b>
	Finds coordinates of $A$ $x = -\frac{7}{2} \Rightarrow y = -\frac{1}{7} \times -\frac{7}{2} - 1 = \left(-\frac{1}{2}\right)$	<b>dM1</b>
	Substitutes $x = -\frac{7}{2}$ , $y = -\frac{1}{2}$ into $y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k \Rightarrow k = \dots$	<b>ddM1</b>
	$k = \frac{5}{4}$ CSO	<b>A1</b>
		<b>(4)</b>
		<b>(12 marks)</b>

**(a)**

**M1:** Finds  $\frac{dy}{dx}$ , look for at least two terms correct. They do not need to be simplified.

**A1:** Correct derivative, need not be simplified. ISW after a correct answer

**(b) Marks cannot be retrospectively awarded from work in (d)**

**M1:** Substitutes  $-\frac{7}{2}$  into their  $\frac{dy}{dx}$  to find the gradient of  $C$  at  $A$

**M1:** Applies perpendicular condition to their gradient to find gradient at  $B$

**dM1:** Equates  $\frac{dy}{dx}$  to the gradient of the normal at  $B$ , depends on first M mark and a changed gradient.

**A1\*:** Reaches the given equation with any correct intermediate line shown following  $\frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} = -\frac{1}{7}$

(c)

**M1:** Any valid method to solve the quadratic, factorisation, completing square, formula or calculator may be used (implied by one correct answer). This may be awarded for work in (b)

**A1:** Correct coordinate  $x_B = \frac{3}{2}$  given with reason. See scheme. The reason should reference the sketch, e.g.

E.g. cannot be  $-\frac{11}{6}$  as that is negative, condone reasons like "because  $B$  is positive"

(d) **Marks cannot be awarded from work in (b), but allow the transfer of answers.** E.g.  $y = -\frac{1}{7}x - 1$

**Explanation of Main method:**

**Find equation for  $l$ , then find coordinates for  $A$  or  $B$ , then sub coordinates into equation for  $C$  to find  $k$**

**M1:** Uses their gradient of  $l$  and intercept  $-1$  to form the equation of  $l$ .

The gradient must a result of a changed  $\frac{dy}{dx}$  at  $x = -\frac{7}{2}$ . It cannot be just made up

**dm1:** Finds the coordinates of either  $A$  or  $B$  using the equation for  $l$  and either  $x_A = -\frac{7}{2}$  or  $x_B = \frac{3}{2}$

FYI  $x = -\frac{7}{2} \Rightarrow y = -\frac{1}{7} \times -\frac{7}{2} - 1 = \left(-\frac{1}{2}\right)$  and  $x = \frac{3}{2} \Rightarrow y = -\frac{1}{7} \times \frac{3}{2} - 1 = \left(-\frac{17}{14}\right)$

**ddm1:** A full method to solve for  $k$ . This involves substituting the coordinates of  $A$  or  $B$  in the equation for

curve  $C$ . E.g. See scheme but can also use  $x = \frac{3}{2}$   $y = -\frac{17}{14}$  into  $y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k \Rightarrow k = \dots$

**A1:** CSO  $k = \frac{5}{4}$

**Explanation of Alt method:**

**Use the  $x$  coordinate for  $A$  or  $B$  in the equation for  $C$  to find the  $y$  coordinate for  $A$  or  $B$  in terms of  $k$ .**

**Then use the gradient and point  $A$  or  $B$  to form an equation for  $l$  in terms of  $k$ . Use the fact that the intercept of  $l$  is  $-1$  to form and solve an equation in  $k$**

<b>Alt I (d)</b>	At $A$ $y = \frac{2}{7}\left(-\frac{7}{2}\right)^3 + \frac{1}{7}\left(-\frac{7}{2}\right)^2 - \frac{5}{2}\left(-\frac{7}{2}\right) + k = \dots \left(= k - \frac{7}{4}\right)$	<b>M1</b>
	Equation of $l$ is $y - \left(k - \frac{7}{4}\right) = -\frac{1}{7}\left(x + \frac{7}{2}\right)$ or $y$ intercept is $-\frac{1}{7} \times \frac{7}{2} + k - \frac{7}{4}$	<b>dm1</b>
	$\Rightarrow y = -\frac{1}{7}x + k - \frac{9}{4} \Rightarrow k - \frac{9}{4} = -1 \Rightarrow k = \dots$	<b>ddm1</b>
	$k = \frac{5}{4}$	<b>A1</b>

**M1:** Substitutes  $x = -\frac{7}{2}$  or  $x = \frac{3}{2}$  into the equation for  $C$  and finds the  $y$  coordinate in terms of  $k$ .

This cannot be scored if they substitute any value for  $y$  (except for the correct value which would mean that we would be using the main method).

FYI at  $B$  the  $y$  coordinate is  $k - \frac{69}{28}$

**dm1:** Uses their gradient for normal at  $A$  (or tangent at  $B$ ) and their  $y$  coordinate to find an equation of the

line  $l$  or to find an expression for the intercept. For use of  $B$  expect  $y - \left(k - \frac{69}{28}\right) = -\frac{1}{7}\left(x - \frac{3}{2}\right)$

**ddM1:** Sets their intercept to  $-1$  and solves for  $k$ .

**A1:**  $k = \frac{5}{4}$

**Explanation of Alt II (d)**

**Find the equation for  $l$  (as in main method) but then equate with the equation for  $C$ . Use the fact that the equation formed has a root of either  $3/2$  or  $-7/2$  to set up and solve an equation in  $k$ .**

**M1:** For an attempt at the equation for  $l$ . Score for  $y = -\frac{1}{7}x - 1$

**dM1:** Equate equation for  $l$  with equation for  $C$  and use the fact that a root of this equation is known.

For example  $-\frac{1}{7}x - 1 = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k \Rightarrow 4x^3 + 2x^2 - 33x + 14k + 14 = 0$

Set  $g\left(\pm\frac{3}{2}\right) = 0$  or  $g\left(\pm\frac{7}{2}\right) = 0$  where  $g(x) = 4x^3 + 2x^2 - 33x + 14k + 14$  to form an equation in  $k$

**ddM1:** As above but sets  $g\left(\frac{3}{2}\right) = 0 \Rightarrow k = \dots$   $g\left(-\frac{7}{2}\right) = 0 \Rightarrow k = \dots$  which must lead to a value for  $k$

**A1:**  $k = \frac{5}{4}$

