Pearson Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Pure Mathematics P3 (WMA13) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol ✓ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- └ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),$ where $|pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q),$ where $|pq|=|c|$ and $|mn|=|a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1. (a)	$\dfrac{5x}{x^2+7x+12}+\dfrac{5x}{x+4}=\dfrac{5x+5x(x+3)}{(x+3)(x+4)}$	M1 A1
	$=\dfrac{5x^2+20x}{(x+3)(x+4)}=\dfrac{5x(x+4)}{(x+3)(x+4)}=\dfrac{5x}{x+3}$ *	A1*
		(3)
(b)	$y=\dfrac{5x}{(x+3)}\Rightarrow xy+3y=5x\Rightarrow 5x-xy=3y$	M1
	$\Rightarrow x=\dfrac{3y}{5-y}$ So $\mathrm{f}^{-1}(x)=\dfrac{3x}{5-x}$	A1
	Domain $0<x<5$	A1
		(3)
(c) (i)	$\mathrm{f}(x)=\dfrac{5x}{(x+3)}\Rightarrow(\mathrm{f}'(x)=)\dfrac{5(x+3)-5x}{(x+3)^2}=\dfrac{15}{(x+3)^2}$	M1 A1
(ii)	States (f is an) increasing function with a suitable reason E.g. Since $(x+3)^2$ is positive	A1
		(3)
		(9 marks)

(a)

M1 Attempts to combine the two fractions using a common denominator.
Allow errors on the numerator but at least one of the terms must have been adapted.
Usual rules apply for factorising the quadratic denominator.
Condone invisible brackets and slips on the numerator when combining the two fractions.
Allow the two fractions to be written separately with the same denominator.

A1 For a correct un-simplified fraction with a quadratic numerator and denominator (which may be implied)

$$\frac{5x}{x^2+7x+12}+\frac{5x}{x+4}=\frac{5x(x+4)}{\ldots}+\frac{5x\left(x^2+7x+12\right)}{\ldots}=\frac{5x^3+40x^2+80x}{\left(x^2+7x+12\right)(x+4)}=\frac{x(5x+20)(x+4)}{\left(x^2+7x+12\right)(x+4)}=\frac{x(5x+20)}{\left(x^2+7x+12\right)}$$

A1* Correctly achieves the given answer of $\dfrac{5x}{x+3}$ showing intermediate steps (cso). Expect to see the two fractions combined and then both the numerator and denominator factorised before cancelling terms to achieve full marks. In the case of forming a cubic numerator and denominator you must see the terms collected before factorising. Bracket errors at some point in their working is A0*

(b)

M1 Attempts to change the subject on $y=\dfrac{5x}{x+3}$ (or $x=\dfrac{5y}{y+3}$). Look for cross multiplication with an attempt to collect terms. Do not follow through on their answer to part (a)

A1 $\quad \mathrm{f}^{-1}(x)=\dfrac{3x}{5-x}$ or $\mathrm{f}^{-1}(x)=\dfrac{-3x}{x-5}$ Must be in terms of x. Condone $\mathrm{f}^{-1}=\ldots$ (or $\mathrm{f}^{-1}=y=\ldots$) but **do not allow** just $y=\ldots$ or $\mathrm{f}^{-1}:\ y=\ldots$

A1 $\quad$ Correct domain $0<x<5$

(c) Mark (i) and (ii) together

M1 $\quad$ Attempts to use the quotient rule or product rule. Look for an expression of the form $\dfrac{A(x+3)-5x}{(x+3)^2}$ or $5x(x+3)^{-1}\to B(x+3)^{-1}\pm 5x(x+3)^{-2}$ where A and B are non-zero constants.

A1 $\quad \left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ or $15(x+3)^{-2}$ or $\dfrac{15}{x^2+6x+9}$ Do not allow $\dfrac{5}{x+3}-\dfrac{5x}{(x+3)^2}$ for this mark.

A1 $\quad$ They achieve $\left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ and states

- (that f is an) **increasing function**
- **since** $(x+3)^2$ **is positive**

Alt methods in (a)

Taking out common factors

M1A1: $\dfrac{5x}{x^2+7x+12}+\dfrac{5x}{x+4}=5x\left\{\dfrac{1+(x+3)}{(x+3)(x+4)}\right\}$ or $=\dfrac{5x}{(x+4)}\left\{\dfrac{1+(x+3)}{(x+3)}\right\}$

Methods where candidates "split up" fractions.

M1 $\quad$ Attempts $\dfrac{5x}{x^2+7x+12}$ OR $\dfrac{5}{x^2+7x+12}$ by partial fractions

A1 $\quad \dfrac{5x}{x+3}-\dfrac{5x}{x+4}+\dfrac{5x}{x+4}$

A1* $\quad \dfrac{5x}{x+3}$ You would expect to see $\dfrac{5x}{x+3}-\dfrac{5x}{x+4}+\dfrac{5x}{x+4}$ before proceeding to the given answer.

Alt method in (b)

M1 $\quad$ Writes $y=\dfrac{5x}{x+3}$ as $y=5\pm\dfrac{k}{x+3}$ and then attempts to make x the subject.

A1 $\quad \mathrm{f}^{-1}(x)=\dfrac{15}{5-x}-3$ or equivalent

A1 $\quad 0<x<5$

Alt method in (c)

M1 $\quad$ Writes $y=\dfrac{5x}{x+3}$ as $y=5\pm\dfrac{k}{x+3}$ and attempts the chain rule to get $\pm\dfrac{C}{(x+3)^2}$

A1 $\quad \left(\mathrm{f}'(x)=\right)\dfrac{15}{(x+3)^2}$ or exact equivalent

A1 States that since $(x+3)^2$ is positive so f is increasing function

Question Number	Scheme	Marks
2 (a)	$\left(\frac{13}{3}, 5\right)$	B1 B1
		(2)
(b)(i) **(ii)**	$\mathrm{f}(x) \ldots 5$ 10	B1 ft B1
		(2)
(c)	Attempts to solve either $16-2x \ldots 3x-13+5 \Rightarrow$ a value or inequality in x Or $16-2x \ldots -3x+13+5 \Rightarrow$ a value or inequality in x Both correct critical values $2, \frac{24}{5}$ Selects inside region for their critical values $2 < x < \frac{24}{5}$	M1 A1 dM1 A1
		(4)
(d)	$a = 4, b = \frac{1}{3}$	B1 ft B1
		(2)
		(10 marks)
Alt(c)	$(11-2x)^2 \ldots (3x-13)^2 \Rightarrow 121-44x+4x^2 \ldots 9x^2-78x+169$ $5x^2-34x+48 \ldots 0 \Rightarrow$ a value or inequality in x $2, \frac{24}{5}$ Selects inside region for their critical values $2 < x < \frac{24}{5}$	M1 A1 dM1 A1

Note – Check the diagram and next to the questions for answers

(a)

B1 One correct coordinate. Allow as $x = \frac{13}{3}, y = 5$ or exact equivalent and condone missing brackets.

B1 Both coordinates correct. Allow as $x = \frac{13}{3}, y = 5$ or exact equivalent and condone missing brackets.

(b)(i)

B1ft $\mathrm{f}(x) \ldots 5$. Allow equivalent correct answers. E.g. $y \ldots 5$, $y \in [5, \infty)$, $\mathrm{f} \ldots 5$ or using set notation. Follow through on their y coordinate in (a). Do not allow "range $\ldots 5$"

(b)(ii)

B1 10

(c)

M1 Attempts to solve a correct equation or inequality. Look for the equation or inequality (ignore the direction) after the modulus signs have been removed allowing slips in their rearrangement.

A1 Correct critical values for $x ... 2, \frac{24}{5}$ or exact equivalent which may be part of an incorrect inequality

dM1 Selects inside region for their critical values or both correct inequalities seen for their values.
Allow this to be written separately and allow the critical values to be included within their region.

A1 $2 < x < \frac{24}{5}$ Allow other exact equivalent forms such as $2 < x < \frac{48}{10}$ or $x \in \left(2, \frac{24}{5}\right)$ or $\frac{24}{5} > x > 2$

The inequalities must be presented together on one line for this mark.

Accept two separate statements such as "$x > 2$ AND $x < \frac{24}{5}$" but not "$x > 2$ or $x < \frac{24}{5}$" or "$x > 2,\ x < \frac{24}{5}$"

(d)

B1ft One correct value. Either $a = 4$ or $b = \frac{1}{3}$ but ft on their part (a) so accept $a = \frac{20}{"5"}$ or $b = "\frac{13}{3}" - 4$

B1 $a = 4, b = \frac{1}{3}$ which may be embedded within $y = a\mathrm{f}(x + b)$

Question Number	Scheme	Marks
3.(a) (i)	$0 = 40 - 30e^{1-0.05t} \Rightarrow 1 - 0.05t = \ln\frac{40}{30}$	M1
	$\Rightarrow (k =)$ awrt 14.2	A1
(ii)	March 1814	A1
		(3)
(b)	Attempts $G = 40 - 30e^{1-0.05\times70} = 37.5$ tonnes	M1 A1
		(2)
(c)	40 (tonnes)	B1
		(1)
		(6 marks)

Mark (a)(i) and (ii) together
(a) (i)

M1 Sets $G = 0$ and proceeds to a linear equation in *t* or *k* using a correct method by taking logs of both sides, but condone slips in the rest of their rearrangement.

A1 $(k =)$ awrt 14.2 or 14.25 but allow the exact value $20\left(1 - \ln\frac{4}{3}\right)$ or other equivalent exact expressions such as $\frac{\ln\frac{4}{3} - 1}{-0.05}$ or $20\left(1 - \ln\frac{4}{3}\right)$ or $20\ln\left(\frac{3}{4}e\right)$. Isw after a correct answer.

(a)(ii)

A1 March 1814 but allow April 1814 following "correct" *k*.
Condone "the third month of 1814" or "the fourth month of 1814" following "correct" *k*
Withhold this mark if they have two answers of which one is incorrect.

(b)

M1 Attempts to find *G* with *t* =70 (condone *t* =69) which may be implied by awrt 37.5
The numbers embedded in the equation is sufficient eg $G = 40 - 30e^{1-0.05\times70}$

A1 awrt 37.5 tonnes (requires units)

(c)

B1 40 (tonnes). Condone $G < 40$ or $G \leqslant 40$. Do not allow $G > 40$

Question Number	Scheme	Marks
4 (a)	$2\sin(\theta-30°)=5\cos\theta \Rightarrow 2\sin\theta\cos30°-2\cos\theta\sin30°=5\cos\theta$	M1
	$\div\cos\theta \qquad \Rightarrow 2\tan\theta\cos30°-2\sin30°=5$	dM1
	$\Rightarrow 2\tan\theta\times\frac{\sqrt{3}}{2}-2\times\frac{1}{2}=5$	A1
	$\Rightarrow \sqrt{3}\tan\theta=6 \Rightarrow \tan\theta=2\sqrt{3}$ *	A1*
		(4)
(b)	Attempts $\arctan 2\sqrt{3}$ ..and then subtracts $20°$	M1
	$\Rightarrow x=\text{awrt } 53.9°, 233.9°$	A1, A1
		(3)
		(7 marks)

(a)

M1 Attempts to use $\sin(\theta-30°)=\sin\theta\cos(\pm30°)\pm\cos\theta\sin(\pm30°)$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 5 of $5\cos\theta$

dM1 Divides by $\cos\theta$ to set up an equation in just $\tan\theta$.

They may collect terms in $\sin\theta$ and $\cos\theta$ before dividing by $\cos\theta$ to set up an equation in just $\tan\theta$

An equation with $\cos30°$ and $\sin30°$ still not processed is acceptable.

A1 Fully correct equation in $\tan\theta$ with the $\cos30°$ and $\sin30°$ processed

($\sqrt{3}\sin\theta=6\cos\theta \Rightarrow \tan\theta=2\sqrt{3}$ is acceptable for both A marks)

(Note If they proceed directly to the final answer from

$\tan\theta=\frac{5+2\sin30°}{2\cos30°} \Rightarrow \tan\theta=2\sqrt{3}$ then maximum M1dM1A0A0 unless

$\tan\theta=\frac{5+1}{\sqrt{3}}$ or equivalent is seen before the final given answer.

A1* Correctly proceeds to given answer.

(b) Answers with no working scores 0 marks

M1 Attempts to find a value for x.

Allow $\arctan 2\sqrt{3}$ ...followed by adding or subtracting $20°$. Which may be implied by

$\tan(x+20)=2\sqrt{3} \Rightarrow x=\arctan(2\sqrt{3})\pm20=...$

Alternatively, attempts to use $\sin(x-10°)=\sin x\cos10°\pm\cos x\sin10°$ within the given equation, divides by $\cos x$ to set up an equation in just $\tan x$ and proceeds to find an angle for x

$\tan x=\frac{5\cos20+2\sin10}{2\cos10+5\sin20} \Rightarrow x=...$

A1 One value provided M1 has been scored. Allow either awrt $54°$ or $234°$ (or in radians awrt 0.94 or 4.08)

A1 $x=\text{awrt } 53.9°, 233.9°$ and no others inside the range provided M1 has been scored. Ignore any angles outside the range. Must be in degrees

Question Number	Scheme	Marks
5 (i)	$\int \frac{8}{(2x-3)^3}\,dx = \frac{-2}{(2x-3)^2}\,(+c)$	M1 A1
	$\int_2^4 \frac{8}{(2x-3)^3}\,dx = \left[\frac{-2}{(2x-3)^2}\right]_2^4 = -\frac{2}{25}+2=\frac{48}{25}$	dM1 A1
		(4)
(ii)	$\int x\left(x^2+3\right)^7\,dx = \frac{1}{16}\left(x^2+3\right)^8+c$	M1 A1
		(2)
		(6 marks)
Alt(i)	Let $u=2x-3$ $\int \frac{8}{u^3}\times\frac{1}{2}\,du = -\frac{2}{u^2}\,(+c)$ $\int_2^4 \frac{8}{(2x-3)^3}\,dx = \left[-\frac{2}{u^2}\right]_1^5 = -\frac{2}{25}+2=\frac{48}{25}$	M1 A1 dM1 A1
Alt(ii)	Let $u=x^2+3$ $\int x\left(x^2+3\right)^7\,dx = \int \frac{u^7}{2}\,du = \frac{u^8}{16}+c = \frac{1}{16}\left(x^2+3\right)^8+c$	M1A1

(i)

M1 Achieves $\frac{A}{(2x-3)^2}$ or equivalent or in the alternative method $\frac{A}{u^2}$

A1 Achieves $\frac{-2}{(2x-3)^2}$ or in the alternative method $-\frac{2}{u^2}$ (which may be unsimplified but the indices must be processed). There is no requirement for the $+c$

dM1 Substitutes 2 and 4 into $\frac{A}{(2x-3)^2}$ or equivalent or 1 and 5 into $\frac{A}{u^2}$ and subtracts either way round. May be implied but M1 must have been scored.

A1 $\frac{48}{25}$ or 1.92 isw after a correct answer

(ii)

M1 Achieves $k\left(x^2+3\right)^8$ or equivalent or in the alternative method ku^8. Alternatively multiplies out the expression and integrates achieving an expression of the form

$\pm...x^{16} \pm...x^{14} \pm...x^{12} \pm...x^{10} \pm...x^{8} \pm...x^{6} \pm...x^{4} \pm...x^{2}$

A1 $\frac{1}{16}\left(x^2+3\right)^8+c$ Must be in terms of x and the + c must be present

Allow $\frac{x^{16}}{16}+\frac{3x^{14}}{2}+\frac{63x^{12}}{4}+\frac{189x^{10}}{2}+\frac{2835x^{8}}{8}+\frac{1701x^{6}}{2}+\frac{5103x^{4}}{4}+\frac{2187x^{2}}{2}+c$ or simplified equivalent

Question Number	Scheme	Marks
6 (i)	$\left(\frac{dy}{dx}=\right)\frac{6x}{x^2-5}-8x$	M1 A1
	Stationary point when $\frac{6x}{x^2-5}-8x=0 \Rightarrow x^2=\frac{23}{4} \Rightarrow x=\frac{\sqrt{23}}{2}$ only	dM1 A1
		(4)
(ii)(a)	$\left(\frac{dy}{dx}=\right)4-24\sin x\cos x$	M1
(b)	$\left(\frac{dy}{dx}=\right)4-12\sin 2x$	dM1 A1
	Maximum gradient = 16	A1
		(4)
		(8 marks)

(i)

M1 For differentiating $3\ln\left(x^2-5\right)\to\frac{Ax}{x^2-5}$ or $\frac{A}{x^2-5}$

A1 $\left(\frac{dy}{dx}=\right)\frac{6x}{x^2-5}-8x$ seen or implied by eg $\frac{6x}{x^2-5}=8x$

dM1 For proceeding from $\left(\frac{dy}{dx}=\right)\frac{Ax}{x^2-5}-Bx=0$ to a quadratic or cubic equation. Eg $6x-8x^3+40x=0$ oe
Do not be too concerned with the mechanics of their rearrangement.

A1 For $\left(x=\right)\frac{\sqrt{23}}{2}$ only provided dM1 has been scored. Solving the quadratic or cubic with a calculator is acceptable. Condone recovery of slips provided the method is correct.
$x=0$ and $x=-\frac{\sqrt{23}}{2}$ must be rejected if found or condone some minimal conclusion eg stating $p=23$

Mark (ii) (a) + (b) together

(ii) Way One

M1 For differentiating using the chain rule with $\sin^2 x\to ...\sin x\cos x$

dM1 .....and then using $\sin 2x=2\sin x\cos x$ to proceed to $\left(\frac{dy}{dx}=\right)A+B\sin 2x$. Condone the slip writing $A+B\sin x$ provided their coefficient of $\sin x\cos x$ has been halved or writing $2\sin\cos x$

A1 $\left(\frac{dy}{dx}=\right)4-12\sin 2x$ or $4+\left(-12\right)\sin 2x$ or $4-12\times\sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(ii) Way Two

M1 Attempts to use $\cos 2x = \pm 1 \pm 2\sin^2 x$ in an attempt to write $y = 4x - 12\sin^2 x$ in terms of $\cos 2x$

dM1 ...and then differentiates $\cos 2x \to k\sin 2x$ to proceed to $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) A + B\sin 2x$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 4 - 12\sin 2x$ or $4 + (-12)\sin 2x$ or $4 - 12 \times \sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(Note some candidates may find the second derivative and set equal to 0 and then substitute in their angle to find the maximum gradient.)

Question Number	Scheme	Marks
7.(a)	$\log_{10} M = 1.93\log_{10} 45 + 0.684 \Rightarrow \log_{10} M = 3.8747$	M1
	$(M =)$ awrt 7500 (kg)	A1
		(2)
(b)	$\log_{10} M = 1.93\log_{10} r + 0.684$ $\log_{10} M = \log_{10} r^{1.93} + 0.684$ $M = 10^{\log_{10} r^{1.93} + 0.684}$ $M = 10^{0.684} \times r^{1.93}$ $M = p\,r^q$ $\log_{10} M = \log_{10} pr^q$ $\log_{10} M = \log_{10} p + \log_{10} r^q$ $\log_{10} M = \log_{10} p + q\log_{10} r$	B1
	Either $p = 10^{0.684}$ or $q = 1.93$	M1
	Both $p =$ awrt 4.83 and $q = 1.93$	A1
		(3)
(c)	"p" is the mass (in kg) of a tree with radius 1 cm	B1
		(1)
		(6 marks)

(a)

M1 Substitutes $r = 45$ in $\log_{10} M = 1.93\log_{10} r + 0.684$ and proceeds to $\log_{10} M = \ldots$ (which must be a numerical value) (Condone the use of $r = 0.45$).
Implied by awrt 7500 for $r = 45$ or awrt 1.0 for $r = 0.45$

A1 $(M =)$ awrt 7500 (kg). Condone 7.5×10^3

Alt (a)

M1 Substitutes $r = 45$ in their $M = p\,r^q$ and finds M.

A1 $(M =)$ awrt 7500 (kg)

(b) Mark (b) and (c) together

B1 A correct proof showing that $\log_{10} M = 1.93\log_{10} r + 0.684 \Leftrightarrow M = p\,r^{q}$. Condone log or lg for $\log_{10}$ but use of natural logs ln is B0

Most are starting with $\log_{10} M = 1.93\log_{10} r + 0.684$. Expect to see the addition (or subtraction) law of logs explicitly used or index law being applied before proceeding to the final answer.

Eg $\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} = \log_{10}\left(4.83 \times r^{1.93}\right) \Rightarrow M = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r + 0.684} \Rightarrow M = 10^{1.93\log_{10} r} \times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r + 0.684} \Rightarrow M = r^{1.93} \times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93\log_{10} r} \times 10^{0.684} = 4.83r^{1.93}$ is B1

$\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} \Rightarrow M = 4.83r^{1.93}$ is B0 (addition law of logs not seen)

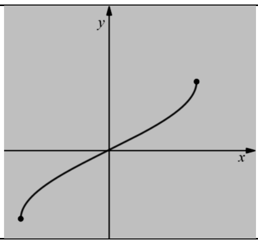
No incorrect working should be seen such as $10^{\log_{10} r^{1.93} + 0.684} = 10^{\log_{10} r^{1.93}} + 10^{0.684} = 10^{\log_{10} r^{1.93}} \times 10^{0.684}$ but condone the omission of the brackets around $\log_{10} 4.83 \times r^{1.93}$ if seen in their working.

M1 Either $p = 10^{0.684}$ or $q = 1.93$ This may be implied by $M = p\,r^{q}$ with a "correct" p or q

A1 Both $p =$ awrt 4.83 and $q = 1.93$ (B0M1A1 can be scored) This may be implied by $M = 4.83\,r^{1.93}$

(c)

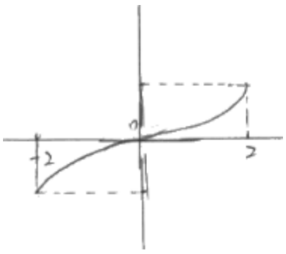
B1 "p" is the mass of a tree with radius 1 cm

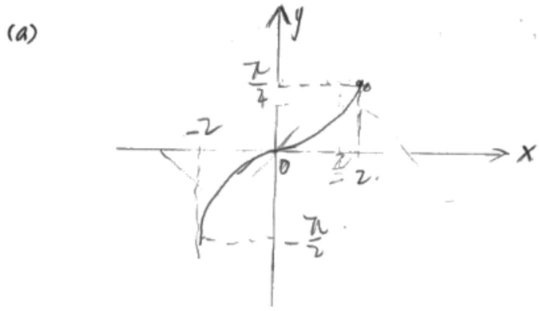
Question Number	Scheme	Marks
8 (a)	 Shape and position	B1 **(1)**
(b)	$x=2\sin y \Rightarrow \left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)2\cos y$ and attempts to use **both** $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\div\dfrac{\mathrm{d}x}{\mathrm{d}y}$ and $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-...x^2}$	M1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{1}{2\sqrt{1-\dfrac{x^2}{4}}}$	A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{1}{\sqrt{4-x^2}}$	A1cso
		(3)
(c)	Substitutes $x=\sqrt{2}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\sqrt{4-x^2}}$ $\left(=\dfrac{\sqrt{2}}{2}\right)$	M1
	Finds the equation of the tangent at P $y-\dfrac{\pi}{4}="\dfrac{\sqrt{2}}{2}"\left(x-"\sqrt{2}"\right)$	dM1
	$y=\dfrac{\sqrt{2}}{2}x-1+\dfrac{\pi}{4}$	A1
		(3)
		(7 marks)
Alt(b)	$y=\arcsin\left(\dfrac{x}{2}\right)\Rightarrow\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{1}{2}\times\dfrac{1}{\sqrt{1-\left(\dfrac{x}{2}\right)^2}}$	M1A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)\dfrac{1}{\sqrt{4-x^2}}$	A1

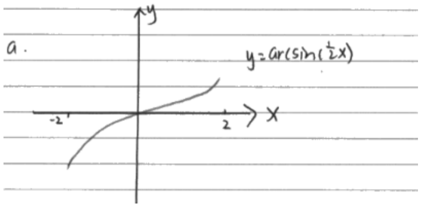
(a)

B1 Correct shape and position: Look for a curve in quadrants 1 and 3 with non zero gradient at the origin and gradient $\to\infty$ at both ends. Ignore values labelled on the axes.
If there is more than one attempt, mark the one which appears in the main body of the work as their fullest attempt. If there is more than one sketch on the same graph, then it must be clearly labelled which is the one to be marked.

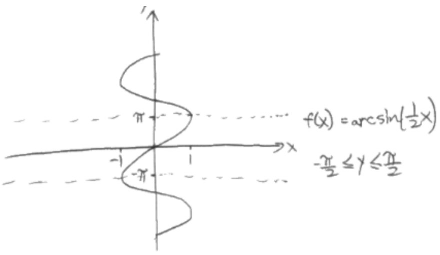
Examples of B1

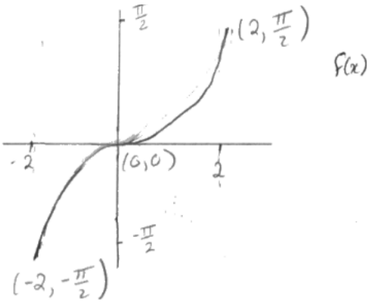






Examples of B0





(b) Note on EPEN it is M1M1A1 which is now marked as M1A1A1

M1 $x = 2\sin y \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \pm k\cos y$ where k is non zero and attempts to use both $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \dfrac{\mathrm{d}x}{\mathrm{d}y}$ and $\sin^2 y + \cos^2 y = 1$ in order to obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. Allow slips with dealing with their "2"

Allow working leading to $\dfrac{\ldots}{\ldots\sqrt{1-\ldots x^2}}$ to score M1.

A1 A correct expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x which may be unsimplified.

eg $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x} =\right)\dfrac{1}{2\sqrt{1-\dfrac{x^2}{4}}}$ or $\dfrac{1}{\sqrt{\dfrac{4}{4}-\dfrac{x^2}{4}}}$ or $\dfrac{1}{2\sqrt{\dfrac{4-x^2}{4}}}$

A1cso $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x} =\right)\dfrac{1}{\sqrt{4-x^2}}$

Alt (b)

M1 $y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = k \times \dfrac{1}{\sqrt{1-\left(\dfrac{x}{2}\right)^2}}$ (where k can equal 1) then A1A1 as above

SC $y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{4-x^2}}$ with no correct intermediate working score as 100.

(c)

M1 Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ at P. See scheme but allow sub of $y=\frac{\pi}{4}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}=\pm k\cos y$

dM1 Full method to find the equation of the tangent at P. Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ may not be exact. If they use $y=mx+c$ they must proceed as far as $c=...$

A1 Correct equation and in the correct form. The coefficient of x and the constant must be exact. Isw

Question Number	Scheme	Marks
9 (a)	$\text{f}(x)=\left(x^3-4x\right)\text{e}^{-\frac{1}{2}x} \Rightarrow \text{f}'(x)=\left(3x^2-4\right)\text{e}^{-\frac{1}{2}x}-\frac{1}{2}\left(x^3-4x\right)\text{e}^{-\frac{1}{2}x}$	M1 A1
		(2)
(b)	$\text{f}'(0)=-4$ so equation of normal is $y=-\frac{1}{-4}x$	M1
	$y=\frac{1}{4}x$	B1
	Sets $\frac{1}{4}\cancel{x}=\cancel{x}\left(x^2-4\right)\text{e}^{-\frac{1}{2}x} \Rightarrow x^2-4=\frac{1}{4}\text{e}^{\frac{1}{2}x}$	M1
	$\Rightarrow x^2=\dfrac{16+\text{e}^{\frac{1}{2}x}}{4} \Rightarrow x=-\frac{1}{2}\sqrt{16+\text{e}^{\frac{1}{2}x}}$ *	A1*
		(4)
(c)	(i) $x_2=-\frac{1}{2}\sqrt{16+\text{e}^{\frac{1}{2}\times -2}}=-2.0229$	M1 A1
	(ii) $(x=)-2.0226$	A1
		(3)
		(9 marks)

(a)

M1 Uses a valid method to differentiate. This could be:

(i) using the product rule on $\text{f}(x)=\left(x^3-4x\right)\text{e}^{-\frac{1}{2}x}$ so score for an expression of the form $(\text{f}'(x)=)\ \pm A\left(x^3-4x\right)\text{e}^{-\frac{1}{2}x}\pm\left(Bx^2\pm C\right)\text{e}^{-\frac{1}{2}x}$. ($A, B, C\neq 0$) Condone the squared missing on the Bx^2 term

(ii) using the quotient rule on $\text{f}(x)=\dfrac{x^3-4x}{\text{e}^{\frac{1}{2}x}}$ so score for an expression of the form

$$(\text{f}'(x)=)\ \frac{\pm\text{e}^{\frac{1}{2}x}\left(Bx^2\pm C\right)-A\text{e}^{\frac{1}{2}x}\left(x^3-4x\right)}{\left(\text{e}^{\frac{1}{2}x}\right)^2}$$ ($A, B, C\neq 0$) Condone the squared

missing on the Bx^2 term or an attempt at (iii) $\text{f}'(x)=uvw'+uv'w+u'vw$ which could look like:

$$(x^2-4)\mathrm{e}^{-\frac{1}{2}x} + x\frac{\mathrm{d}\left((x^2-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x} = \pm...(x^2-4)\mathrm{e}^{-\frac{1}{2}x} \pm x(...x)\mathrm{e}^{-\frac{1}{2}x} \pm ...x(x^2-4)\mathrm{e}^{-\frac{1}{2}x}$$

A1 Correct $\mathrm{f}'(x)$ but may be unsimplified. Isw after a correct unsimplified expression

$$\left(3x^2-4\right)\mathrm{e}^{-\frac{1}{2}x} - \frac{1}{2}\left(x^3-4x\right)\mathrm{e}^{-\frac{1}{2}x} \text{ or } (x^2-4)\mathrm{e}^{-\frac{1}{2}x} + x\frac{\mathrm{d}\left((x^2-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x} = (x^2-4)\mathrm{e}^{-\frac{1}{2}x} + x(2x)\mathrm{e}^{-\frac{1}{2}x} - \frac{1}{2}x(x^2-4)\mathrm{e}^{-\frac{1}{2}x}$$

(b) Note on EPEN it is M1A1M1A1 but we are marking this M1B1M1A1

M1 Full method to find the equation of the normal through *O*.

Look for an attempt at $\mathrm{f}'(0)$ followed by the equation $y = -\frac{1}{\mathrm{f}'(0)}x$

B1 Equation of normal is $y = \frac{1}{4}x$ (seen or implied) (which may follow an incorrect $\mathrm{f}'(x)$ from part (a))

M1 Equates their $y = \frac{1}{4}x$ (which must be a straight line through the origin) with $\mathrm{f}(x) = x\left(x^2-4\right)\mathrm{e}^{-\frac{1}{2}x}$, divides through or factorises out the *x* term and attempts to make x^2 (or allow $4x^2$) the subject

A1* Full proof showing all steps. There is no requirement to justify the – sign. Note that A1* cannot be scored if A0 in part (a), unless they restart in (b).

(c)(i)

M1 Substitutes $x = -2$ into the iteration formula and finds x_2. May also be implied by –2.0228 or –2.0229

A1 awrt –2.0229

(ii)

A1 $(x =) -2.0226$ correct to 4 dp

Question Number	Scheme	Marks
10.(a)	$(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 2x$	
	Uses $\cos 4x = 2\cos^2 2x - 1 \Rightarrow (1+2\cos 2x)^2 = 1+4\cos 2x+2\cos 4x+2$	M1
	$= 3+4\cos 2x+2\cos 4x$	A1
		(2)
(b)	$a = \dfrac{2\pi}{3}$	B1
	$\int 3+4\cos 2x+2\cos 4x \, \mathrm{d}x = 3x+2\sin 2x+\frac{1}{2}\sin 4x$	M1 A1 ft
	Area $= \left[3x+2\sin 2x+\frac{1}{2}\sin 4x\right]_0^{\frac{2\pi}{3}} = 2\pi - \frac{3}{4}\sqrt{3}$	dM1 A1
		(5)
		(7 marks)

(a)

M1 Attempts to multiply out $(1+2\cos 2x)^2 = 1+...\cos 2x+...\cos^2 2x$ and use $\cos 4x = 2\cos^2 2x - 1$ to obtain $(1+2\cos 2x)^2$ in the form $p+q\cos 2x+r\cos 4x$.

Condone slips in the rearrangement of $\cos 4x = 2\cos^2 2x - 1$ but it must be clear that the identity was correct originally otherwise M0.

Beware of candidates who write $(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 4x$ which is M0A0

A1 $3+4\cos 2x+2\cos 4x$

(b)

B1 Deduces that $a = \dfrac{2\pi}{3}$ (allow 120° for this mark). If more than one angle is found, then look for which one is substituted into their integrated expression.

M1 Integrates $q\cos 2x + r\cos 4x \to \pm...\sin 2x \pm ...\sin 4x$

A1ft Integrates $p+q\cos 2x+r\cos 4x \to px + \frac{q}{2}\sin 2x + \frac{r}{4}\sin 4x$ unsimplified where *p*, *q* and $r \neq 0$

dM1 Substitutes 0 and $a = \dfrac{2\pi}{3}$ (or awrt 2.09) (or equivalent) into a valid function (M1 must have been scored) and subtracts either way around. Note q, $r \neq 0$ but they do not need a *px* term.

Also allow $a = \dfrac{\pi}{3}$ (or awrt 1.05) or $a = \dfrac{4\pi}{3}$ (or awrt 4.19) *a* must be in radians to evaluate correctly.

This mark cannot be scored without a value for a. You do not have to explicitly see 0 substituted in and their answer may imply a correct substitution into their integrated expression.

A1 $2\pi - \frac{3}{4}\sqrt{3}$ or simplified equivalent

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