Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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| | |

**Pearson Edexcel**
**International Advanced Level**

Centre Number | Candidate Number

# Friday 8 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F1

**You must have:**
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶







Pearson

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**1.** (a) Show that the equation $4x - 2\sin x - 1 = 0$, where $x$ is in radians, has a root $\alpha$ in the interval [0.2, 0.6]

**(2)**

(b) Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which $\alpha$ lies.

**(3)**

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**Question 1 continued**

Q1

**(Total 5 marks)**

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**2.** Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}\mathrm{i}$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

**(1)**

Given that $x = 4$ is also a root of the equation,

(b) find the value of $p$ and the value of $q$.

**(4)**

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**Question 2 continued**

Q2

**(Total 5 marks)**



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**3.** The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of $k$ for which **M** is singular.

**(2)**

Given that **M** is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of $k$.

**(2)**

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**Question 3 continued**

Q3

**(Total 4 marks)**

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**4.** The equation $2x^2 + 5x + 7 = 0$ has roots $\alpha$ and $\beta$

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

**(3)**

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

**(5)**

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**Question 4 continued**



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**Question 4 continued**

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**Question 4 continued**

Q4

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**5.** (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n}(r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers $n$.

**(5)**

(b) Hence show that

$$\sum_{r=n+1}^{2n}(r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where $a$ and $b$ are integers to be determined.

**(2)**

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**Question 5 continued**





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**Question 5 continued**

**(Total 7 marks)**

Q5





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**6.** The complex number $z$ is defined by

$$z = -\lambda + 3\text{i} \quad \text{where } \lambda \text{ is a positive real constant}$$

Given that the modulus of $z$ is 5

(a) write down the value of $\lambda$

**(1)**

(b) determine the argument of $z$, giving your answer in radians to one decimal place.

**(2)**

**In part (c) you must show detailed reasoning.**

**Solutions relying on calculator technology are not acceptable.**

(c) Express in the form $a + \text{i}b$ where $a$ and $b$ are real,

(i) $\dfrac{z + 3\text{i}}{2 - 4\text{i}}$

(ii) $z^2$

**(5)**

(d) Show on a single Argand diagram the points $A$, $B$, $C$ and $D$ that represent the complex numbers

$$z,\ z^*,\ \frac{z + 3\text{i}}{2 - 4\text{i}} \text{ and } z^2$$

**(3)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

Q6

**(Total 11 marks)**

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**7.** The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle $T$ onto triangle $T'$

Given that the area of triangle $T$ is 23 cm$^2$

(a) determine the area of triangle $T'$

**(2)**

The point $P$ has coordinates $(3p + 2, 2p - 1)$ where $p$ is a constant. The transformation represented by **A** maps $P$ onto the point $P'$ with coordinates $(17, -18)$

(b) Determine the value of $p$.

**(2)**

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

**(2)**

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

**(3)**

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**Question 7 continued**



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**Question 7 continued**

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**Question 7 continued**

**(Total 9 marks)**

Q7



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**8.** The hyperbola $H$ has Cartesian equation $xy = 25$

The parabola $P$ has parametric equations $x = 10t^2$, $y = 20t$

The hyperbola $H$ intersects the parabola $P$ at the point $A$

(a) Use algebra to determine the coordinates of $A$

**(3)**

The point $B$ with coordinates $(10, 20)$ lies on $P$

(b) Find an equation for the normal to $P$ at $B$

Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be determined.

**(5)**

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola $H$

**(6)**

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**Question 8 continued**



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**Question 8 continued**

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**Question 8 continued**

Q8

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**9.** (i) A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1$$

**(6)**

(ii) $f(n) = 2^{n+2} + 3^{2n+1}$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 7

**(6)**

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

Q9

**(Total 12 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**