Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level
in Pure Mathematics P1 (WMA11) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

- **B** marks are unconditional accuracy marks (independent of M marks)

- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol ⩓ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ✱ The answer is printed on the paper or ag- answer given
   - ⌐ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),$ where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q),$ where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use <u>correct</u> formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c,$ $q \neq 0,$ leading to $x = \ldots$

Method marks for differentiation and integration:
1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1.	$$\int\left(\frac{8}{3}x^3 - \frac{1}{2}x^{-\frac{1}{2}} - 5\right)dx = \frac{8}{3}\times\frac{x^4}{4} - \frac{1}{2}\times 2x^{\frac{1}{2}} - 5x + c$$	M1 A1
	$$= \frac{2}{3}x^4 - x^{\frac{1}{2}} - 5x + c$$	A1 A1
		(4 marks)

M1 For raising a power of x by 1 seen at least once (i.e. $x^n \to x^{n+1}$) or $-5 \to -5x$.
The index does not have to be processed for this mark.

Award following incorrect manipulations eg $-\dfrac{1}{2\sqrt{x}} \to ...x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$

A1 For two of $\dfrac{8}{3}\times\dfrac{x^4}{4}$, $-\dfrac{1}{2}\times 2x^{\frac{1}{2}}$, $-5x^1$ correct (unsimplified).
This may be implied by a correct simplified answer. May be seen on different lines. Indices must be processed. Ignore any spurious notation for any of the A marks.

A1 For two of $\dfrac{2}{3}x^4$, $-x^{\frac{1}{2}}, -5x$ correct and in simplest form. Accept forms such as $\dfrac{2x^4}{3}$ and $-\sqrt{x}$

Condone $-5x^1$, $-1x^{\frac{1}{2}}$ but not $-\dfrac{5x}{1}$. Correct terms may be seen on different lines. Allow $\dfrac{2}{3}$ to be written as $0.\dot{6}$ or 0.6666….. only (must be exact)

A1 Fully correct **on one line**, and simplified with $+c$. Accept simplified equivalents (see above)

Question Number	Scheme	Marks
2 (a)	$3^{3x} = \left(3^x\right)^3 = y^3$	B1 (1)
(b)	$\dfrac{1}{3^{x-2}} = \dfrac{1}{3^x \times 3^{-2}} = \dfrac{9}{y}$	M1 A1 (2)
(c)	$\dfrac{81}{9^{2-3x}} = \dfrac{9^2}{9^{2-3x}} = 9^{2-(2-3x)} = 9^{3x} = 3^{6x} = y^6$	M1 A1 (2)
		(5 marks)

Note: Correct answer in any part implies full marks for that part

(a)

B1 y^3 Condone $(y)^3$. Ignore once correct answer seen.

(b)
M1   For correct application of the addition/subtraction law so award for eg:

$$\frac{1}{3^x \times 3^{-2}} \text{ or } \frac{1}{3^x \div 3^2} \text{ or } \frac{1}{\left(\dfrac{3^x}{3^2}\right)} \text{ or } \frac{1}{y \times 3^{-2}} \text{ or } \frac{3^2}{3^x} \text{ or sight of 9 or } \frac{1}{y} \text{ oe}$$

A1   For $\dfrac{9}{y}$ or $9y^{-1}$ but NOT expressions that still contain $\div$ or fractions within fractions or $3^2 y^{-1}$. Ignore once correct answer seen.

(c)

M1   For simplifying the indices to expressions of the form $9^{..}$ or $3^{..}$ (not as a denominator) so award for $9^{3x}, \left(9^x\right)^3, 3^{6x}, \left(3^x\right)^6$, or $k \times y^6$, $k \neq 0$ which must come from correct working. Also allow unsimplified equivalent expressions of the final answer eg $\dfrac{1}{y^{-6}}$ as long as it in terms of y

A1 y^6 only (not eg $\dfrac{1}{y^{-6}}$). Ignore once correct answer seen. Condon $(y)^6$

Note: In all parts they may work from $y = 3^x$ and manipulate both sides to the given answer which is acceptable.

Eg part (b): $y = 3^x \Rightarrow y \times 3^{-2} = 3^{x-2}$ (M1) $\Rightarrow \dfrac{1}{y \times 3^{-2}} = \dfrac{1}{3^{x-2}} \Rightarrow \dfrac{9}{y} = \dfrac{1}{3^{x-2}}$ (A1)

Eg part (c): $y = 3^x \Rightarrow y^{-6} = 3^{-6x} \Rightarrow 81y^{-6} = 3^{4-6x}$ (M1) $\Rightarrow 81y^{-6} = 9^{2-3x} \Rightarrow \dfrac{81}{9^{2-3x}} = y^6$ (A1)

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Question Number	Scheme	Marks
3.(a)	Attempts $\left(\dfrac{dy}{dx}=\right) 2x+3$ at $x=3$	M1
	At $x=3$ gradient of tangent $=9$	A1
		(2)
(b)	$\left(y_Q=\right) (3+h)^2+3(3+h)-2$	B1
	Gradient $PQ=\dfrac{(3+h)^2+3(3+h)-2-16}{3+h-3}=\dfrac{9h+h^2}{h}=9+h$	M1 A1
		(3)
(c)	States as $h\rightarrow 0$ Gradient $PQ\rightarrow 9$ = Gradient of tangent	B1
		(1)
		(6 marks)

(a)

M1   Attempts to find the value of $\left(\dfrac{dy}{dx}=\right) ax+3, \quad a>0$ at $x=3$. Look for 3 to be substituted into the expression and proceeding to a value.

A1   9 (Answer only scores both marks)

(b)

B1 $\left(y_Q=\right) (3+h)^2+3(3+h)-2$ (seen or implied)

M1   Attempts $\pm\dfrac{y_Q-16}{x_Q-3}$ condoning slips, but must be a genuine attempt at y_Q. Condone lack of brackets if implied by later working.

A1 $9+h$ with no errors seen and not originating from methods in calculus. This expression may immediately follow a correct simplified expression of $y_Q=h^2+9h+16$

(c)

B1   States as $h\rightarrow 0$ Gradient $PQ\rightarrow 9$ = Gradient of tangent (oe)

They must have achieved $9+h$ in (b) and 9 in (a)

There should be reference to "limit" or "as h tends to 0" (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these contraints.

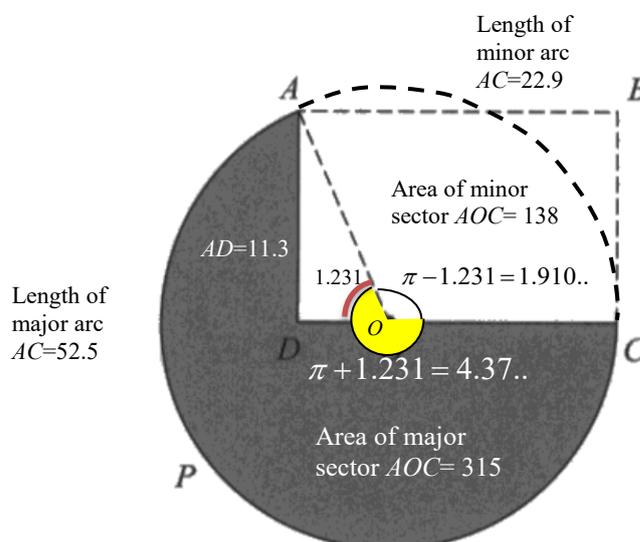| Question Number	Scheme	Marks
4 (a)	States or uses $\cos AOD = \dfrac{4}{12} \Rightarrow$ angle $AOD = 1.231$ *	M1 A1*
		(2)
(b)	Attempts $\dfrac{1}{2}r^2\theta$ with $r = 12$ and $\theta = \pi \pm 1.231$ or 1.231	M1
	Attempts area $AOD = \dfrac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$ oe $(22.627...)$	M1
	Attempts sector $-$ triangle $= \dfrac{1}{2} \times 12^2 \times (\pi + 1.231) - \dfrac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$ $(314.8....) \ - \ (22.627...)$	ddM1
	or Attempts circle-sector-triangle $\pi \times 12^2 - \dfrac{1}{2} \times 12^2 \times (\pi - 1.231) - \dfrac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$ $452.38... \quad - \quad 137.562.... \quad - \quad 22.627......$ $= $ awrt 292.2 (m^2)	A1
		(4)
(c)	Attempts $s = r\theta$ with $r = 12$ and $\theta = \pi \pm 1.231$ or 1.231	M1
	Attempts $P = 16 + \sqrt{12^2 - 4^2} + 12(\pi + 1.231)$ oe	dM1
	$=$ awrt 79.8 (m)	A1
		(3)
		(9 marks)
Alt(a)	$AD = \sqrt{12^2 - 4^2} = 8\sqrt{2}$ $\cos AOD = \dfrac{12^2 + 4^2 - (8\sqrt{2})^2}{2 \times 12 \times 4} \Rightarrow$ angle $AOD = \cos^{-1}\left(\dfrac{1}{3}\right) = 1.231$	M1A1*

(a)

M1   Attempts $\cos AOD = \dfrac{4}{12} \Rightarrow$ angle $AOD = ...$ via a correct method.

Alternatively attempts to find AD using Pythagoras' theorem correctly **and** uses the appropriate sine or tangent. They may attempt to use the cosine rule with all three side (see alt(a)).
Other candidates are finding angle OAD and using angles in a triangle to find angle AOD.

Do not award this mark for candidates who attempt to use 1.231 and compare with $\dfrac{4}{12}$

A1*   Achieves angle $AOD = 1.231$ following a valid method and at least one step of working shown.

Minimum acceptable $\cos^{-1}\left(\dfrac{4}{12}\right) = 1.231$ or $\cos AOD = \dfrac{4}{12} \Rightarrow$ angle $AOD = 1.231$

Only withhold this mark if
- awrt 1.231 radians is not achieved following a correct method. FYI sight of 1.23095… is likely to imply both marks)
- they work in degrees but fail to achieve awrt 70.5˚ **before** proceeding to 1.231 radians

We are going to condone on this occasion any rounding that may be written as part of their intermediate steps as long as they achieve awrt 1.231. Ignore any omission of brackets.

(b)

M1   Attempts $\frac{1}{2}r^2\theta$ with $r=12$ and an allowable θ. ($\theta = \pi \pm 1.231$ or 1.231).

         $\angle AOC =$ awrt 1.911 which may appear as ($\pi - 1.231$) or ($2\pi - (\pi + 1.231)$) (minor sector)

         $\angle AOC =$ awrt 4.373 which may appear as ($\pi + 1.231$) or ($2\pi - (\pi - 1.231)$) (major sector)

         The embedded values in an expression is sufficient for this mark. (awrt 315, awrt 88.6 or awrt 138 also implies this mark. Condone candidates who round 1.231 or use eg awrt 4.4 or awrt 1.9 radians.

M1   Correct method to find area of triangle AOD. Eg $\frac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$, $\frac{1}{2} \times "4" \times 12 \sin 1.231$, awrt 22.6.

         The angle may be in degrees (70.5….) and condone using awrt 1.2 radians.

         Also allow an alternative method finding the area of the rectangle – area of trapezium. Allow errors in their method to finding "4" but r must be 12.

ddM1   Full method to find the correct area $\frac{1}{2} \times 12^2 \times (\pi + 1.231) - \frac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$ oe. The embedded

         values in an expression is sufficient for this mark and the angle may be awrt 4.4. It is dependent on the two previous method marks.

         Alternatively they may find the area by

         Area of circle – area of minor sector – area of triangle

$$\pi \times 12^2 - \frac{1}{2} \times 12^2 \times (\pi - 1.231) - \frac{1}{2} \times "4" \times \sqrt{12^2 - "4"^2}$$

A1   awrt 292.2 (m^2)

(c)

M1   Attempts $s = r\theta$ with $r = 12$ and an allowable θ. ($\theta = \pi \pm 1.231$ or 1.231 as above in part (b)). The embedded values in an expression is sufficient for this mark. (awrt 52.5 or awrt 14.8 or awrt 22.9 can imply this mark. Condone candidates who round 1.231 or use eg awrt 4.4 or awrt 1.9 radians).

dM1   Full method to find the perimeter $P = 16 + \sqrt{12^2 - "4"^2} + 12(\pi + 1.231)$ oe . The embedded values in an expression is sufficient for this mark and the angle may be awrt 4.4 or awrt 1.9.

         Alternatively they may find the perimeter P via circumference $-$ minor arc $+ 16 + "8\sqrt{2}"$

$$P = 16 + \sqrt{12^2 - "4"^2} + 24\pi - 12(\pi - 1.231)$$

A1   awrt 79.8 (m)

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Question Number	Scheme	Marks
5. (a)	$20x^3 - 50x^2 - 30x = 0 \Rightarrow 10x(2x^2 - 5x - 3) = 0$ $\Rightarrow 10x(2x+1)(x-3) = 0$ $\Rightarrow x = 0, -\dfrac{1}{2}, 3$	M1 A1, A1 **(3)**
(b)	Sets or implies $(y+3)^{\frac{1}{2}} = 0$ or $-\dfrac{1}{2}$ or 3 Full method to find y $y = 6$ $y = -3,\ 6$	B1ft M1 A1ft, A1 **(4)** **(7 marks)**

(a) **Note the question says using algebra so answers only scores 0 marks.**

M1   For attempting to factorise. They may do this by cancelling/factorising out $\ldots x$ to achieve an expression of the form $Dx(Ax^2 + Bx + C)$ where $A, B, C \neq 0$ (but could be 1) and $D = 1, 2, 5$ or 10
If they only have the resulting quadratic $(2x+1)(x-3)$ they must state somewhere that $x = 0$ (usually as one of their answers)
Equally they may proceed straight to a fully factorised form eg. $x(2x+1)(x-3)\ (=0)$
We will condone on this occasion $x(x+\frac{1}{2})(x-3)$

A1   Two of $x = 0, -\dfrac{1}{2}, 3$ **following M1 awarded** .

A1   All of $x = 0, -\dfrac{1}{2}, 3$ They do not have to be stated on one line together. Withhold if any additional solutions.

(b)

B1ft   Sets $(y+3)^{\frac{1}{2}} =$ any of their solutions from (a) or this may be implied by later work on one of their solutions from (a). Note $(y+3) = 0 \Rightarrow y = -3$ is insufficient for the B mark but is acceptable for the final A mark (see below)

M1   Full method to find a value for y from a non-zero solution for x.
Scored for **squaring** and **subtracting 3**
Allow this to be scored for squaring and subtracting 3 from a negative value as well

A1ft $y = 6$. Follow through on their positive solution from part (a)

A1   Both of $y = -3, 6$ with no incorrect working seen.   If the solution $-\dfrac{11}{4}$ is found it must be discounted by the candidate (eg crossed out/rejected).

Beware that part (b) is hence so they must use their solutions from (a) and be careful regarding the -3 solution.

$(y+3)^{\frac{1}{2}} = 0 \Rightarrow y = -3$ and $(y+3) = 0 \Rightarrow y = -3$ is acceptable for the final A mark BUT

$(y+3)^{\frac{1}{2}} = 0 \Rightarrow y + 3 = \sqrt{0} \Rightarrow y = -3$ is an incorrect method so A0

Give benefit of the doubt where it is unclear but send to review if unsure.

Question Number	Scheme	Marks
6.(a)	Attempts to find the gradient of $3x - 4y + 20 = 0 \Rightarrow y = \underline{\underline{\dfrac{3}{4}}}x + 5$	M1
	Equation l_2 is $y - 0 = "\dfrac{3}{4}"(x-8) \Rightarrow 3x - 4y - 24 = 0$ oe	M1, A1
		(3)
(b)	$P = \left(-\dfrac{20}{3}, 0\right)$, $Q = (0, 5)$	B1
	Area $PQRS = PR \times OQ = \left(8 + "\dfrac{20}{3}"\right) \times "5" = \dfrac{220}{3}$	M1, A1
		(3)
(c)	$\overrightarrow{QR} = \begin{pmatrix} 8 \\ -5 \end{pmatrix} \Rightarrow S = \left(-\dfrac{20}{3} + 8, 0 - 5\right) = \left(\dfrac{4}{3}, -5\right)$	M1, A1
		(2)
		(8 marks)
Alt(c)	Solve their $y = "\dfrac{3}{4}x - 6"$ with their $"y = -\dfrac{5}{8}\left(x + \dfrac{20}{3}\right)"$	
	$\dfrac{3}{4}x - 6 = -\dfrac{5}{8}\left(x + \dfrac{20}{3}\right) \Rightarrow \dfrac{11}{8}x = \dfrac{11}{6} \Rightarrow x = ...$	M1
	$S = \left(\dfrac{4}{3}, -5\right)$	A1
		(2)

If you see any unusual approaches which may involve vectors and matrices then please send to review if you are unsure how to mark.

(a)

M1   For an attempt to find the gradient of $3x - 4y + 20 = 0$
      Look for an attempt to rearrange $3x - 4y + 20 = 0$ and make y the subject.
      Expect to see $\pm 4y = ...$ followed by $y = ...$ or equivalent if they do not achieve the correct value.

M1   For using the same gradient as their gradient of l_1 with $R(8,0)$ to form a linear equation.
      If they use $y = mx + c$ they must proceed as far as $c = ...$ starting from a correct equation with their gradient of l_1 and the coordinates correctly substituted in.
      Some candidates may set $3x - 4y + c = 0$ and substitute $(8,0)$ to find c which is M1M1.

A1 $3x - 4y - 24 = 0$ or any integer multiple. All terms must be on one side of an equation $= 0$.
      Correct equation implies full marks and it must be seen in (a) to score the A mark. (They may make errors in the manipulation of l_1 which is condoned).

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Mark parts (b) and (c) together.

(b)

B1 $P = \left(-\dfrac{20}{3}, 0\right)$, $Q = (0, 5)$. This may be implied by subsequent calculations or seen on a diagram

M1   A full attempt to find the area of parallelogram $PQRS = PR \times OQ = \left(8 + "\dfrac{20}{3}"\right) \times "5"$ using their P and

their Q and proceeding to find a value for the area. Condone attempts which appear to be calculating parallelogram $PQSR$.

Candidates may use the "Shoelace" algorithm i.e. $2 \times \dfrac{1}{2} \begin{vmatrix} 8 & 0 \\ 0 & 5 \\ -\dfrac{20}{3} & 0 \\ 8 & 0 \end{vmatrix} = 2 \times \dfrac{1}{2} \times \left(8 \times 5 - -\dfrac{20}{3} \times 5\right)$

Candidates may attempt to find the area of the parallelogram using the determinant

eg $\dfrac{20}{3}\underline{i} + 5\underline{j}$ and $8\underline{i} - 5\underline{j}$ Area $= \left| \det \begin{bmatrix} \dfrac{20}{3} & 5 \\ 8 & -5 \end{bmatrix} \right| = \left| \dfrac{20}{3} \times -5 - 8 \times 5 \right| = \left| -\dfrac{220}{3} \right| \Rightarrow$ Area $= \dfrac{220}{3}$

A1 $\dfrac{220}{3}$ oe (condone awrt 73.3)

(SC Sign slip on the coordinate Q to give $P = \left(-\dfrac{20}{3}, 0\right)$, $Q = (0, -5) \Rightarrow$ Area of parallelogram =

$\dfrac{220}{3}$ **scores 011)**

Some candidates who do part c) and form parallelogram $PQSR$ to find $S = \left(\dfrac{44}{3}, 5\right)$ may then

attempt b) and proceed to the correct answer. This can still score M1 A1 in (b)

(c)

M1   A full attempt to find both the coordinates of S. Examples include:
- $\overrightarrow{OP} + \overrightarrow{QR}$ or equivalent using a vector approach.
- An attempt to solve their $y = \dfrac{3}{4}x - 6$ with their $y = -\dfrac{5}{8}\left(x + \dfrac{20}{3}\right)$
- An attempt to equate the length RS with PQ (or PS with RQ) to produce an equation that is solved simultaneously with l_2 or with each other (see next page)
- $y_s = -5$ from rotational symmetry consideration and proceeds to finding x
- It is also acceptable to score full marks following a sketch. Send to review if you find any which you think are credit-worthy.

In all cases they should proceed to find the coordinates of S but condone arithmetical slips in their working. If in doubt send to review.

A1 $\left(\dfrac{4}{3}, -5\right)$. Allow alternatives eg $x = \ldots$, $y = \ldots$ Condone (awrt $1.33, 5$)

SC: For those students who find $S = \left(\dfrac{44}{3}, 5\right)$ score SC 10

Additional guidance: some examples of methods for (c) using the lengths between points
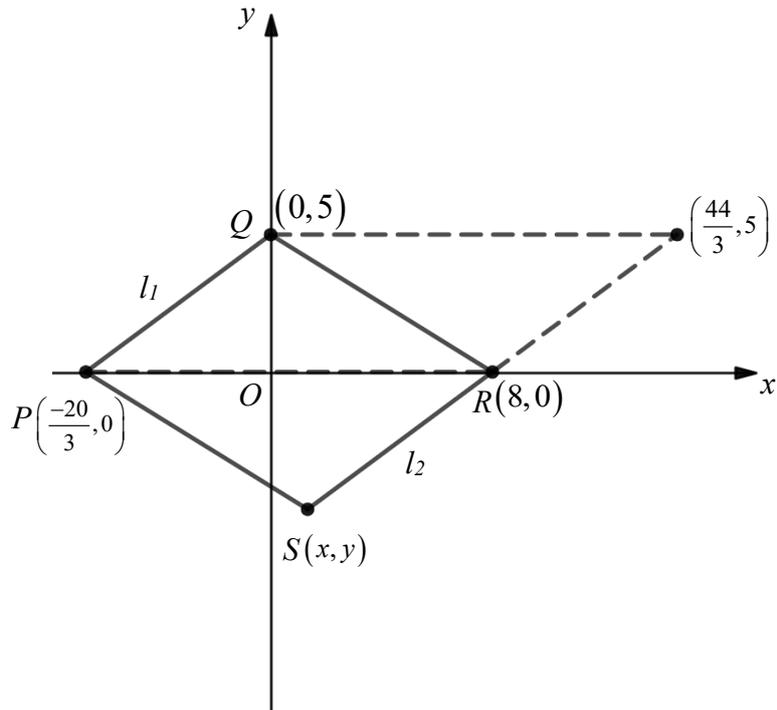
1. Equating RS: $(8,0)$ to (x,y) with PQ: $\left(\dfrac{-20}{3},0\right)$ to $(0,5)$ and solving simultaneously with l_2

   Look for $(x-8)^2 + y^2 = \dfrac{625}{9}$ and $3x - 4y - 24 = 0$ proceeding to $x = \dfrac{4}{3}$, $y = -5$

2. Equating PS: $\left(\dfrac{-20}{3},0\right)$ to (x,y) with RQ: $(8,0)$ to $(0,5)$ and solving simultaneously with l_2

   Look for $(x--\dfrac{20}{3})^2 + y^2 = 89$ and $3x - 4y - 24 = 0$ proceeding to $x = \dfrac{4}{3}$, $y = -5$

3. Equating RS: $(8,0)$ to (x,y) with PQ: $\left(\dfrac{-20}{3},0\right)$ to $(0,5)$ and equating PS: $\left(\dfrac{-20}{3},0\right)$ to (x,y) with RQ: $(8,0)$ to $(0,5)$ and solving simultaneously

   Look for $(x-8)^2 + y^2 = \dfrac{625}{9}$ and $(x--\dfrac{20}{3})^2 + y^2 = 89$ which proceeds to

   $9x^2 - 144x + 9y^2 - 49 = 0$ and $9x^2 + 120x + 9y^2 - 401 = 0 \Rightarrow 264x = 352 \Rightarrow x = \dfrac{4}{3}$, $y = -5$

Question Number	Scheme	Marks
7. (a)(i)	$P = (90°, 3)$	M1, A1
(ii)	$Q = (540°, 0)$	B1
		(3)
(b)	$(270°, 4)$	M1, A1
		(2)
		(5 marks)

Note we condone the absence of the degree symbol in this question.

(a)(i)

M1   For one value in the coordinate pair of $(90°, 3)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone $(…,90˚)$ or $(3,…)$ Allow in radians.

A1   For both of $(90°, 3)$. Allow $x = 90$, $y = 3$

(a)(ii)

B1 $Q = (540°, 0)$ Allow $x = 540$, $y = 0$

(b)

M1   For one value in the coordinate pair of $(270°, 4)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone $(…,270˚)$ or $(4,…)$. Allow in radians.

A1   For both of $(270°, 4)$. Allow $x = 270$, $y = 4$

For answers missing the brackets or radians(or both) in all parts then penalise the first time the A or B mark is due

FYI (a)(i) $\left(\frac{\pi}{2}, 3\right)$ (a)(ii) $Q = (3\pi, 0)$, (b) $\left(\frac{3\pi}{2}, 4\right)$ would score M1 A0(first error) B1 M1 A1

For all correct values the wrong way round $(3, 90°), (0, 540°) (4, 270°)$ **SC M1A0B0M1A1**

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Question Number	Scheme	Marks
8.	Equates $y = k(2x-1)$ and $y = x^2 + 2x + 11 \Rightarrow k(2x-1) = x^2 + 2x + 11$	M1
	$\Rightarrow x^2 + (2-2k)x + 11 + k \ (=0)$	A1
	Attempts "$b^2 - 4ac$"...$0 \Rightarrow (2-2k)^2 - 4(11+k)$...$0$	
	and proceeds to critical values	M1
	Critical values of $(k =)$ $5, -2$	A1
	No roots so $b^2 - 4ac < 0$ so choose inside region	M1
	$-2 < k < 5$	A1
		(6)
		(6 marks)

M1   Attempts to equate $y = k(2x-1)$ and $y = x^2 + 2x + 11 \Rightarrow k(2x-1) = x^2 + 2x + 11$

A1 $x^2 + (2-2k)x + 11 + k \ (=0)$ oe

        Correct quadratic or correct values of a, b and c. The terms should be collected on one side of the equation and the x terms must have been collected together in a bracket. Condone a missing "=0". This mark may be implied by values of a, b and c used in later work.

M1   Attempts "$b^2 - 4ac$" where $a = \pm 1$, $b = \pm 2 \pm 2k$, $c = \pm 11 \pm k$ to achieve a 3TQ and proceeds to find critical values. Condone arithmetical slips in the rearrangement to a 3TQ. Usual rules for solving a quadratic apply. You may see "$b^2...4ac$".

        FYI "$b^2 - 4ac$" $= 4k^2 - 12k - 40$ (Be careful that this may result from incorrect working which loses the A marks that follow)

A1   Correct critical values $(k =)$ $5, -2$ **MUST HAVE COME FROM CORRECT WORKING**

M1   Finds inside region for their critical values. May be awarded for eg $-2 \leqslant k \leqslant 5$ or $-2 \leqslant k < 5$. Allow this mark if another variable is used.

A1 $-2 < k < 5$ oe such as $k \in (-2, 5)$ cso

        Note that $-2 < x < 5$ is A0

Question Number	Scheme	Marks
9.	$\dfrac{4x^2+9}{2\sqrt{x}} = \dfrac{4x^2}{2\sqrt{x}} + \dfrac{9}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \dfrac{9}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left(\dfrac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \dfrac{9}{4}x^{-\frac{3}{2}}$	M1 A1
	$\left(\dfrac{dy}{dx}=\right)\ 3x^{\frac{1}{2}} - \dfrac{9}{4}x^{-\frac{3}{2}} = 0 \Rightarrow x^2 = \dfrac{3}{4} \Rightarrow x = \dfrac{\sqrt{3}}{2}$	M1 A1
		(6)
		(6 marks)
Alt(I)	Quotient rule	
	$u = 4x^2+9, u' = 8x, v = 2\sqrt{x}, v' = x^{-\frac{1}{2}}$	M1A1
	$\left(\dfrac{dy}{dx}=\right)\ \dfrac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x}$	M1A1
	$\left(\dfrac{dy}{dx}=\right)\ \dfrac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \dfrac{3}{4} \Rightarrow x = \dfrac{\sqrt{3}}{2}$	M1A1
Alt(II)	Product rule	
	$u = 4x^2+9, u' = 8x, v = \dfrac{1}{2}x^{-\frac{1}{2}}, v' = \dfrac{1}{4}x^{-\frac{3}{2}}$	M1A1
	$\left(\dfrac{dy}{dx}=\right)\ (4x^2+9)\times\dfrac{1}{4}x^{-\frac{3}{2}} + 8x\times\dfrac{1}{2}x^{-\frac{1}{2}}$	M1A1
	$\left(\dfrac{dy}{dx}=\right)\ \dfrac{2\sqrt{x}\times 8x - (4x^2+9)\times x^{-\frac{1}{2}}}{4x} = 0 \Rightarrow x^2 = \dfrac{3}{4} \Rightarrow x = \dfrac{\sqrt{3}}{2}$	M1A1

M1   Attempts to divide by $2\sqrt{x}$. Award for one correct term (including $\dfrac{9}{2x^{\frac{1}{2}}}$). Allow if they combine the two terms with a common denominator of 2, but the indices must have been processed.

If they use the quotient rule it is for selecting u and v and attempting to differentiate. Look for $u = 4x^2 + 9, u' = ...x, v = 2\sqrt{x}, v' = ...x^{-\frac{1}{2}}$. If they use the product rule then it is for

$$u = 4x^2 + 9, u' = 8x, v = \frac{1}{2}x^{-\frac{1}{2}}, v' = \frac{1}{4}x^{-\frac{3}{2}}$$

A1 $2x^{\frac{3}{2}} + \dfrac{9}{2}x^{-\frac{1}{2}}$ which may be left unsimplified but the indices must be processed. Using the product or quotient rule it is for having correct u' and v'.

M1   Attempts to differentiate the expression written as a sum. Award for one power decreasing by one on one of their terms following through their sum and the indices must have been processed. Cannot be awarded if they just differentiate top and bottom of the fraction.

Using the quotient rule look for expressions of the form $\left(\dfrac{dy}{dx} = \right) \dfrac{2\sqrt{x} \times ...x \pm (4x^2 + 9) \times ...x^{-\frac{1}{2}}}{4x}$

Using the product rule look for expressions of the form $\left(\dfrac{dy}{dx} = \right) (4x^2 + 9) \times ...x^{-\frac{3}{2}} + ...x \times \dfrac{1}{2}x^{-\frac{1}{2}}$

A1 $\left(\dfrac{dy}{dx} = \right) 3x^{\frac{1}{2}} - \dfrac{9}{4}x^{-\frac{3}{2}}$ which may be left unsimplified but the indices must be processed.

Using the quotient rule award for $\left(\dfrac{dy}{dx} = \right) \dfrac{2\sqrt{x} \times 8x - (4x^2 + 9) \times x^{-\frac{1}{2}}}{4x}$.

Using the product rule award for $\left(\dfrac{dy}{dx} = \right) (4x^2 + 9) \times \dfrac{1}{4}x^{-\frac{3}{2}} + 8x \times \dfrac{1}{2}x^{-\frac{1}{2}}$

M1   Sets $\dfrac{dy}{dx} = 0$ and proceeds to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$ following a derivative in the form
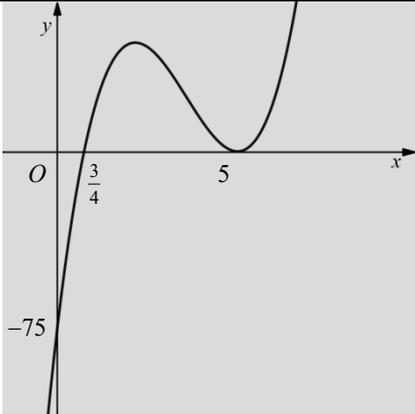
$$\left(\frac{dy}{dx} = \right) Ax^{\frac{1}{2}} - Bx^{-\frac{3}{2}}, \quad A, B > 0 \text{ (which may be unsimplified).}$$

Using the quotient rule or product rule look to proceed from one of the forms above to $x^{\pm 2} = ...$ or $x^{\pm 4} = ...$

Rounded versions of the answer do not imply this mark. Do not be too concerned by the mechanics of their arrangement but we must see some attempt to carry out algebraic manipulation proceeding to the required form.

A1 $(x =) \dfrac{\sqrt{3}}{2}$ or exact equivalent cso (Note that a correct exact answer can imply the final M1A1 but a rounded answer with no working such as awrt 0.87 is M0A0)

Withhold the final mark if $-\dfrac{\sqrt{3}}{2}$ is not rejected.

Question Number	Scheme	Marks
10.(a)	Shape for +ve x^3	B1
	Cuts x-axis at $\left(\dfrac{3}{4},0\right)$ and meets at $(5,0)$	B1
	Crosses y-axis at $(0,-75)$	B1
		(3)
(b)(i)	$(x=)$ 3, 20	B1ft
(ii)	$(p=)75$	B1ft
		(2)
(c) (i)	$(g(x)=)$ $\left(4(x+1)-3\right)(x+1-5)^2 = (4x+1)(x-4)^2$	M1 A1
(ii)	16	B1
		(3)
		(8 marks)

(a)

For any of the coordinates if there are contradictions between the graph and the text then the graph takes precedence.

B1 Shape for a positive cubic, any position, with one maximum and one minimum. Condone no axes for this mark. Condone cubic curves which have a cusp like appearance at the minimum point.

B1 For a graph crossing at $\left(\dfrac{3}{4},0\right)$ and meeting at $(5,0)$. The graph should not stop or cross at $(5,0)$.

Allow just the x values instead of the full coordinates marked on the axes or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.**

B1 For a graph crossing the y-axis at $(0,-75)$

Allow the y value to be marked instead of the full coordinate or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.** Do not condone 75 marked on the negative y-axis.

Note: in both (b)(i) and (ii) for the follow through the graph takes precedence.
(b)(i)

B1ft $(x =)$ 3, 20 and no others Follow through on $4\times$ their x intercepts.
        Allow $(3,0)$ $(20,0)$ ignore $(0,-75)$

(b)(ii)

B1ft $(p =)$ 75 Follow through on their y intercept. Allow if $(y =)$ $f(x)+75$ seen. Do not allow $y = 75$

Mark (i) and (ii) together

(c)(i)

M1   Attempts $g(x) = \left(4(x+1)-3\right)(x+1-5)^2$ condoning slips. Either award for sight of $x+1$ embedded in
       the equation or award for sight of an expression of the form $(4x+a)(x-4)^2$
       Alternatively they expand $(4x-3)(x-5)^2 = 4x^3 + Px^2 + Qx + R$, where P,Q and $R \neq 0$, and then
       replace x with $x+1$ to achieve $4(x+1)^3 + P(x+1)^2 + Q(x+1) + R$ (condoning slips).

A1 $(g(x) =)$ $(4x+1)(x-4)^2$ or simplified equivalent so accept $4x^3 - 31x^2 + 56x + 16$

(c)(ii)
B1   16 but accept $(0,16)$.
       Note that they may attempt $f(1) = (4\times1-3)(1-5)^2 = 16$

Question Number	Scheme	Marks
11 (a)	Gradient of normal $= \dfrac{1}{4}$	B1
	Equation of normal $(y+50) = "\dfrac{1}{4}"(x-4) \Rightarrow y = \dfrac{1}{4}x - 51$	M1 A1
		(3)
(b)	$(f''(x) =) \ \dfrac{6}{\sqrt{x^3}} + x = 6x^{-\frac{3}{2}} + x \Rightarrow f'(x) = -12x^{-\frac{1}{2}} + \dfrac{1}{2}x^2 + k$	M1 A1
	Substitutes $x = 4, \ f'(x) = -4 \Rightarrow k = -6$	dM1 A1
	$(f'(x) =) \ -12x^{-\frac{1}{2}} + \dfrac{1}{2}x^2 - 6 \Rightarrow (f(x) =) \ -24x^{\frac{1}{2}} + \dfrac{1}{6}x^3 - 6x + d$	dM1 A1ft
	Substitutes $x = 4, \ f(x) = -50 \Rightarrow d = \dfrac{34}{3}$	dddM1
	$(f(x) =)-24x^{\frac{1}{2}} + \dfrac{1}{6}x^3 - 6x + \dfrac{34}{3}$	A1
		(8)
		(11 marks)

Mark (a) and (b) together

(a)

B1   Deduces that the gradient of the normal is $\dfrac{1}{4}$

M1   Attempts to find the equation of a line passing through $P(4,-50)$ with a **changed** gradient. Allow one sign slip on a coordinate so either $(y+50)$ or $(x-4)$ must be correct. If they use $y = mx + c$ then at least one of the coordinates must be correctly substituted in and they must proceed as far as $c = ...$

A1 $y = \dfrac{1}{4}x - 51$

(b)

M1   Attempts to integrate $\dfrac{6}{\sqrt{x^3}} + x$ with one index correct. Either $...x^{-\frac{1}{2}}$ or $...x^2$

A1 $(f'(x)) = -12x^{-\frac{1}{2}} + \dfrac{1}{2}x^2 + k$ (unsimplified) with or without the $+k$

dM1   Substitutes $x = 4, \ f'(x) = -4$ into an integrated form (with $+k$) and proceeds to find the value of k
This is dependent on the first M1.

A1 $(f'(x)=) -12x^{-\frac{1}{2}} + \dfrac{1}{2}x^2 - 6$ (unsimplified) which may be implied

dM1   Dependent upon the first M. It is for integrating 'again' with one index correct. Either $...x^{\frac{1}{2}}$ or $...x^3$

A1ft $(f(x)=) -24x^{\frac{1}{2}} + \dfrac{1}{6}x^3 - 6x + d$ (unsimplified) following through ONLY on their k (allow kx) and with or without d

dddM1 Dependent upon all three previous Ms.

Both "k" and "d" must have been added although condone calling them both c.

This mark is scored for using $x = 4,\ f(x) = -50$ in an attempt to find 'd'. Do not be concerned by the mechanics of their rearrangement.

A1 $\left(f(x)=\right)\ -24x^{\frac{1}{2}} + \dfrac{1}{6}x^3 - 6x + \dfrac{34}{3}$ or **exact equivalent expressions**. Eg Do not allow $\dfrac{1}{6}$ to be written as 0.167 but condone $-6x^1$. The indices must have been processed and the terms must all be on one line including $\dfrac{34}{3}$. isw after a correct expression.